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ABSTRACT

The papers presented in the strand on "Women and Mathematics" at the 1978 Annual Meeting of the National Council of Teachers of Mathematics form the core of this monograph. Two commentaries synthesizing the presentations and offering additional suggestions for action are also included. The papers consider sex-related differences in mathematics achievement, mathematics problems, affective components, mathematics anxiety, sexism in textbooks, mathophia, factors leading to success and the role of the mathematics education community and others in promoting change. A selected bibliography is also given. (MS)

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PERSPECTIVES ON
WOMEN AND MATHEMATICS

edited by

Judith E. Jacobs

ERIC Clearinghouse for Science, Mathematics
and Environmental Education
College of Education
The Ohio State University
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Columbus, Ohio 43212

1978

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TABLE OF CONTENTS

FOREWORD v

PREFACE
Shirley Hill vii

SEX-RELATED DIFFERENCES IN MATHEMATICS ACHIEVEMENT: WHERE AND WHY?
Elizabeth Fennema 1

ARE MATHEMATICS PROBLEMS A PROBLEM FOR WOMEN AND GIRLS?
Ann K. Schonberger 21

WHY SUSIE CAN'T--OR DOESN'T WANT TO--ADD
Grace M. Burton 35

SEX, SEXISM AND ANXIETY IN MATHEMATICS
John Ernest 59

SEXISM IN MATHEMATICS TEXTBOOKS
William H. Nibbelink and Matadial Mangru . . . 65

MATHOPHOBIA: A MINI-COURSE FOR THE MATHOPHOBIC
Ruth Afflack 73

PROGRAMS TO COMBAT MATH AVOIDANCE
Rita May Liff 101

A CLINICAL APPROACH TO REDUCING MATH ANXIETY
Sheila Tobias 113

FACTORS LEADING TO SUCCESS--PRESENT AND FUTURE
Patricia Lund Casserly 119

CHANGE: THE ROLE OF THE MATHEMATICS EDUCATION COMMUNITY
Judith E. Jacobs 125

MULTIPLE LEVELS FOR CHANGE
Lucy W. Sells 137

COMMENTS ON THE WOMEN AND MATHEMATICS STRAND, 1978 NCTM ANNUAL MEETING
Judith S. Shoemaker 143

SOME ADDITIONAL COMMENTS ON THE WOMEN AND MATHEMATICS STRAND, 1978 NCTM ANNUAL MEETING
Dora Helen Skypek 147

CONTRIBUTORS 153

SELECTED BIBLIOGRAPHY ON WOMEN AND MATHEMATICS 155

FOREWORD

The low rate of participation of girls and women in the study of mathematics has become a national issue of great concern. Those concerned are not just the traditional feminists who have raised many questions about sex-role stereotypes, but include members of the traditional education community in such places as the National Institute of Education, Office of Education and various schools and professional associations across the nation. The National Council of Teachers of Mathematics, through its program committee for its 56th Annual Meeting held in San Diego, California, 12-15 April 1978, provided a forum for the discussion of many of the questions associated with the task of increasing the number of girls and women who pursue mathematical studies or mathematics-related careers.

The sessions in the strand on Women and Mathematics at the San Diego meeting were very well attended. The papers presented at these sessions form the core of this monograph. Two commentaries have also been prepared which synthesize the material presented at the San Diego meeting and offer additional suggestions for action. This collection is a vehicle for sharing the best thinking available on the problem of changing the rate of participation of girls and women in the study and use of mathematics with those who did not attend the NCTM meeting.

Many individuals have contributed to the development of this monograph. Louis Cohen, who chaired the program committee for the San Diego meeting, was instrumental in placing the Women and Mathematics strand on the program. Jon Higgins of the ERIC/SMEAC Clearinghouse encouraged and supported the preparation of this monograph. Jonathan Crook of George Mason University provided valuable assistance in the preparation of the bibliography. However, this publication would not have been possible without the cooperation of the individuals who prepared these papers. Their willingness to share these through this monograph is greatly appreciated. In this way the discussion of and progress towards the solution of the issues connoted by the phrase Women and Mathematics can continue.

Judith E. Jacobs
George Mason University
Fairfax, Virginia
July 1978

PREFACE

At the 56th Annual Meeting of the National Council of Teachers of Mathematics in San Diego, there was a palpable air of excitement surrounding one of the topic strands. This program strand was organized around issues concerning the mathematics learning of girls and young women. The section meetings were filled to overflowing.

This was not the first instance in which NCTM had demonstrated concern for the urgent need to attack the very real problems recognized in the mathematical education of females. There had been sections devoted to these topics at NCTM meetings for several years and program booklets from both the 54th and 55th Annual Meetings show increasing numbers of such sections. But there had not been the focus, the emphasis, the continuity and concentration made possible by the topic strand at the San Diego meeting.

We are at a stage where we now know a great deal about the reality of the problem statistically. No objective person who reads the enrollment figures for high school mathematics classes can fail to appreciate that extremely powerful societal factors operate to discourage girls from continuing in mathematics study. And there is no doubt that the failure to go beyond one or two years of mathematics in high school closes many, many doors in both employment and continuing academic progress.

On the positive side, we can say that we are making strides. More and more programs provide models for successful attack upon some of the barriers, research provides increasing useful data, sexism is being eliminated gradually from the textbooks and, most important, increasing attention is being paid to the fact that there is a problem. In short, the overt manifestations of sexism in mathematics education are being attacked and gradually overcome.

But perhaps the most serious influences, those subtle messages that society conveys in myriad, difficult-to-pinpoint ways, represent the area in which we in education must concentrate our greatest efforts. The attitudes and conditioning related to the individual female's role and the perceived need and appropriateness of mathematical study to that role, are developed early and are the result of innumerable interrelated experiences, both in school and out. Thus the critical character of the elementary school and middle or junior high school experiences, the influence of teachers, counselors and parents becomes apparent.

Professional educators must, in my view, make both individual and organizational commitments to the formidable task of assuring truly equal opportunity for girls and young women to achieve in mathematics. This entails the elimination of psychological as well as institutional barriers. The commitments must be deliberate and specific, supported by systematic and adequately financed programs. It is in the interests of both simple justice and practical economic productivity to do so.

Shirley Hill, President
National Council of Teachers
of Mathematics 1978

SEX-RELATED DIFFERENCES IN MATHEMATICS ACHIEVEMENT: WHERE AND WHY?*

Elizabeth Fennema

Mathematics educators have used sex as a variable in research concerned with mathematics achievement for a number of years and many summaries of mathematics achievement have been published which include information about comparative learning of mathematics by females and males. Basically, all reviews published before 1974 concluded that while there might not be a sex-related difference in mathematics achievement in young children, male superiority was always evident by the time learners reached upper elementary or junior high school. In addition, males were definitely superior on higher level cognitive tasks. E.g., "The evidence would suggest to the teacher that boys will achieve higher than girls on tests dealing with mathematical reasoning" (Glennon and Callahan, 1968, p. 50). "From junior high school and beyond . . . boys now surpass girls in studies involving science and mathematics" (Suydam & Riedesel, 1969, p. 129). "Sex differences in mathematical abilities are of course, present at the kindergarten level and undoubtedly earlier" (Aiken, 1971, p. 203).

The literature reviews published since 1974 do not show the same consensus about male superiority that was evident previously. In a 1974 review which synthesized information from 36 studies, the conclusion was that there were no sex-related differences in elementary school children's mathematics achievement, and little evidence that such differences exist in high school learners. However, there was some indication that males excelled in higher level cognitive tasks and females in lower level cognitive tasks (Fennema, 1974). Callahan and Glennon (1975) agreed with this conclusion while Maccoby and Jacklin (1974) in a highly quoted review disagreed. They stated that one "sex difference that [is] fairly well established . . . is that boys excel in mathematical study" (pp. 351-352). From these reviews it is evident that currently there is not consensus on whether sex-related differences in mathematics achievement exists.

The question of whether or not there are sex-related differences in mathematics is more complicated than it first appears. While there is no doubt that many more males than females are involved in post high school mathematics study and in adult occupations that involve mathematics, what has been unclear is whether this unequal representation of females and males in post high school in the study and use of mathematics is due to females' less adequate learning of mathematics or to deliberate choice by females not to study mathematics. Both of these issues will be addressed here.

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Sex-Related Differences in Mathematics Achievement

In order to clarify the reality of sex-related differences in mathematics achievement, four major studies of sex-related differences in mathematics achievement will be specifically noted: Project Talent, the National Longitudinal Study of Mathematical Abilities (NLSMA), the National Assessment of Educational Progress (NAEP), and the Fennema-Sherman studies. In addition, some studies from other cultures will be briefly reviewed as well as the Stanley study of mathematically precocious youth and scores on college entrance examinations.

Data for Project Talent were gathered about 1960 (Flanagan et al., 1964). This study assessed the mathematics achievement (among many other things) of a random sampling of high school students in the United States (n 440,000). The data indicated that in grade 9 sex-related differences in mathematics achievement were negligible but by 12th grade males tended to do better. The mean difference at the 12th grade, while statistically significant, appears to have little educational significance (approximately one item). No attempt was made to control the number of mathematics courses subjects had taken previously. Higher percentages of males undoubtedly had taken more mathematics courses and males were more apt to say they were preparing for a career which needed mathematics. Undoubtedly a population of males with more mathematical background was being compared with a population of females with less mathematical background.

In 1975 a follow up to the 1960 Project Talent study was done. Data were collected from approximately 1800 students in grades 9-11 in seventeen of the original schools (Flanagan, 1976). After careful statistical checks on reliability of the comparisons and adjustments for any change in school SES the following were concluded: (See Table 1). 1) While the mathematics test scores were fairly stable from 1960 to 1975, the differences between females and males had been reduced; 2) On computation tasks, male scores had declined 17 percent and female scores 11 percent with the female mean score being 8.2 points higher than the male mean score; 3) Quantitative reasoning scores declined 8 percent for each sex with females scoring .6 of an item lower in 1975. It is difficult, after carefully examining these data from Project Talent, to conclude that males' mathematics achievement was much higher than that of females in 1960 or 1975.

Support for the belief that females do not achieve as well as males in mathematics could come from the NLSMA data which were gathered during 1962-67. In these multitudinous studies, sex was used as a control variable. Analyses were done independently by sex whenever significant sex by any other variable interaction was found. Unfortunately, the results from these studies have been inadequately reported and interpreted making the knowledge they could contribute to the area under consideration largely unavailable. However, a summary statement says: "Differences favoring girls were for variables at the comprehension level (the lowest cognitive level tested) and the differences favoring the boys were for variables at the application and analysis level" (Wilson, 1972, p. 94).

Table 1

PROJECT TALENT*

Comparisons of 1960 and 1975 Mathematics Results

	MALES			FEMALES		
	Raw Score		10th Grade Percentile			10th Grade Percentile
	1960	1975	Difference	1960	1975	Difference
Quantitative Reasoning	8.5	7.8	-8%	8.0	7.2	-8%
Mathematics	10.5	10.7	+2%	9.9	10.3	+3%
Computation	25.7	18.7	-17%	30.8	26.9	-11%

*Abstracted from: Flanagan, J. C. Changes in school levels of achievement: Project TALENT 10 and 15 year retests. _Educational Researcher_, 1976, _5_(8), 9-12.

The directors of this federally-financed program abrogated their responsibility to females when they followed the above remarks with this statement: "Interpretation and comment on this pattern will be left to persons involved in the women's liberation movement" (Wilson, 1972, p. 95). The number of mathematics courses which had been taken previously by the subjects in the NLSMA studies was controlled so the conclusion reached undoubtedly was statistically valid in 1967. What is unknown is the size of the differences between the mean female and male performance scores and the educational significance of that difference.

Results from the 1972-73 mathematics data collection of the National Assessment of Educational Progress (NAEP) have received much publicity and one sentence has been widely quoted: "In the mathematics assessment, the advantage displayed by males, particularly at older ages can only be described as overwhelming" (Mullis, 1975, p. 7). Inspection of these data (Table 2) confirms that males did outperform females at ages 17 and 26-35. However, at ages 9 and 13 differences were minimal and sometimes in favor of females. The problem of comparable populations is a concern here also. The population was selected by sophisticated random sampling techniques with no control for educational or mathematical background. Since males have traditionally studied mathematics more years than have females, once again a population of males with more background in mathematics was being compared with a population of females with less background in mathematics. At ages 9 and 13 when the educational and mathematical background was similar, the achievement of both sexes was also similar.

The Fennema-Sherman Study, data for which were collected in 1975-76, investigated mathematics achievement in grades 6-12 (Fennema and Sherman, 1977; Sherman and Fennema, 1977; Fennema and Sherman, 1978). This National Science Foundation sponsored study investigated a variety of levels of mathematics learning as well as cognitive and affective variables hypothesized to be related to differential mathematics achievement by females and males. The results of this study have wide generalizability because of the diverse, carefully selected sample. In grades 9-12 (n = 1233) with subjects whose mathematics backgrounds were carefully controlled, significant differences in achievement in favor of males (approximately two items) were found in two of four schools. In grades 6-8 (n = 1330) significant differences were found in favor of females in a low cognitive level mathematical task in one of four tested school areas. In another of the four school areas, significant differences were found in favor of males in a high cognitive level mathematical task.

One other measure of mathematics achievement should be noted, the differential evident between the sexes on the SAT (Scholastic Aptitude Test), a college entrance examination. Normally administered to high school seniors, it has a Verbal Component and a Mathematics Component. According to the publishers of the test, the mathematics required is that which is taught in grades 1-9. Women have, over a period of years, scored lower than men on this test. However, a look at some trends is interesting. "In 1960 the mathematical component means were 465 for women, 520 for men. Twelve years later, the average for women was virtually unchanged, but the average for men had dropped by 14 points (to

506)" (Wirtz, 1977, p. 16). Although the Advisory Panel appointed to review the SAT score declines (Wirtz, 1977) concludes that one reason the scores declined from 1962-1970 was that more women were taking the test, the data do not confirm that women's scores dropped.

Once again, however, conclusions about male superiority are being drawn from populations which have studied different amounts of mathematics. Even though the mathematics required for the SAT may be taught before disparity in enrollment between the sexes is evident, continued use of such mathematics in advanced high school classes undoubtedly aids one in solving items of the type included in the SAT.

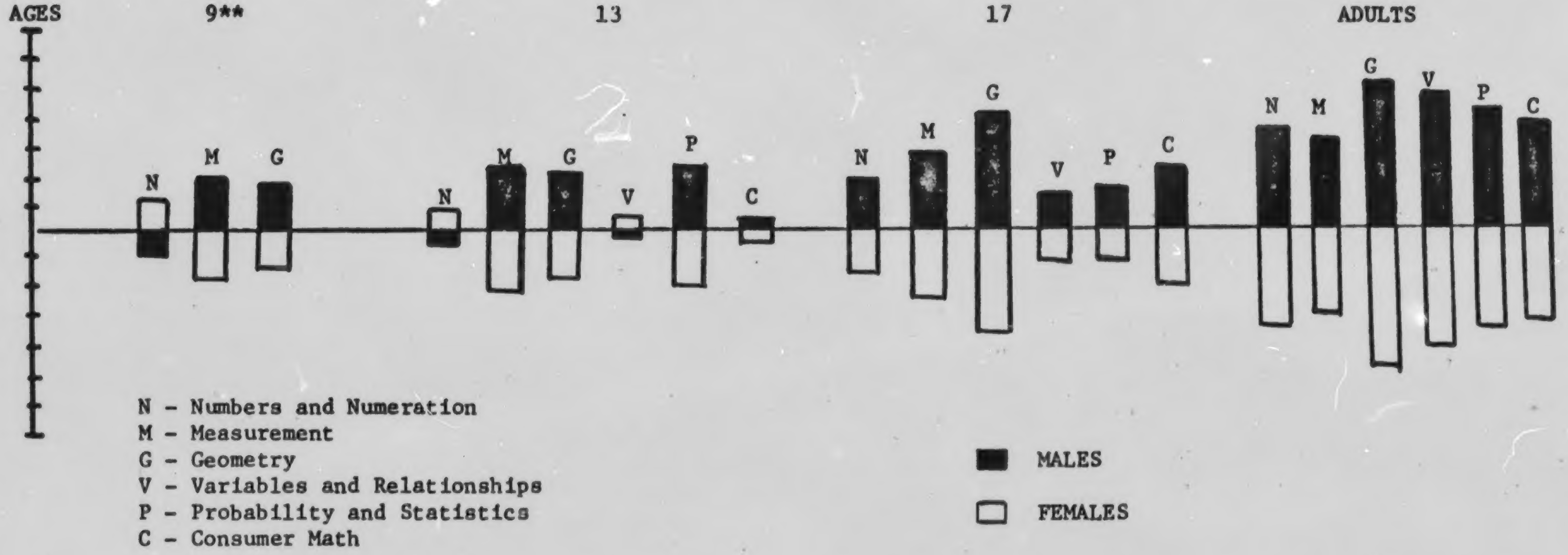
A different perspective on sex-related differences in mathematics achievement is noted if one examines performances of highly precocious males and females. In the Stanley Study of Mathematically Precocious Youth, many males outperformed any female. E.g., in the 1973 talent search junior high school youth who had scored above the 98th percentile on previously given subtests of standardized achievement tests were asked to volunteer to be tested on a college entrance examination. Seven percent of the boys scored higher than any girl and the boys' mean score was significantly higher than the girls' mean score (Fox, 1976).

In summary what can be concluded about sex-related differences in mathematics learning in the United States in 1978 is: 1) There are no sex-related differences evident in elementary school years. This is at all cognitive levels from computation to problem solving. This conclusion has been accepted for a number of years. 2) After elementary school years, differences do not always appear. 3) Starting at about the 7th grade, if differences appear, they tend to be in the males' favor, particularly on tasks involving higher level cognitive skills. 4) There is some evidence that sex-related differences in mathematics learning in high schools may not be as large in 1978 as they were in previous years. 5) Conclusions reached about male superiority have often been gathered from old studies or studies in which the number of mathematics courses taken was not controlled. Therefore, a better mathematically educated group of males was being compared to a group of females who had participated in less mathematics education. In reality, what was being compared were not females and males but students who had studied mathematics 1-3 years in high school with students who had studied mathematics 2-4 years in high school.

An examination of cross cultural differences in mathematics performance is interesting. In Australia, female superiority on problem solving and computation tasks in grades 5-8 is reported while males are reported to have performed at high levels in space tasks (Clements and Watanawaha, 1977). However, Keeves (1973) reports that male superiority over females in mathematics achievement was found within all ten countries which participated in the First International Study of Educational Achievement.

Table 2

Median Difference in Performance Between Males and Nation and Females and Nation on Mathematics Content Areas*



*National Mean

**Mullis, I.V.S. *Educational achievement and sex discrimination*. Denver, Colorado: National Assessment of Educational Progress, 1975.

Sex-related Differences in the Study of Mathematics

There are sex-related differences in the studying of mathematics. This is indicated by females choosing not to enroll in mathematics courses in high school and by the paucity of females in university mathematics courses. Undoubtedly, the most serious problem facing those concerned with equity in mathematical education for the sexes is ensuring that females continue their study of mathematics. In support of this statement consider some data from Wisconsin. During the 1975-76 academic year, while approximately the same number of females and males were enrolled in Algebra, in the advanced courses many more males were enrolled (see Table 3).

Although only symptomatic of the effects of many variables, electing not to study mathematics in high school beyond minimal or college requirements is the cause of many females' nonparticipation in mathematics-related occupations. The one variable which can be positively identified as causing sex-related differences in mathematics learning is the differential number of years females and males spend formally studying and using mathematics. Such a simplistic explanation of such an important problem seems too good to be true. However, this author believes strongly that if the amount of time spent learning mathematics is somehow equated for females and males, educationally significant sex-related differences in mathematics performance will disappear.

Table 3[c]

Number of Males and Females Enrolled in Wisconsin in Mathematics Courses[a]

Course	Males	Females
Algebra[b]	41,404	41,579
Geometry	20,937	20,280
Algebra II	11,581	9,947
Pre-Calculus	3,234	1,917
Trigonometry	4,004	2,737
Analytic Geometry	1,752	970
Probability/Statistics	1,113	581
Computer Mathematics	3,396	1,481
Calculus	611	262

[a] Data obtained from Wisconsin Department of Public Instruction Enrollment Statistics, 1975-76.

[b] Students enrolled in one-year course and two-year course.

[c] Konsin, M.A. Enrollment in Wisconsin high school mathematics classes by sex during 1975-76. Unpublished manuscript, University of Wisconsin-Madison, 1977.

Related Factors

While it is interesting to know about comparative mathematics achievement of females and males, it is more important to gain understanding of why so many males leave educational institutions knowing a great deal more mathematics than females know. In order to gain this understanding selected cognitive, affective and educational variables will be discussed.

Cognitive Variables

"Mathematics is essentially cognitive in nature; and the principal distinguishing goals or objectives of mathematics instruction are (and should be) cognitive ones" (Weaver, 1971, p. 263). Since mathematics is a cognitive endeavor, the logical place to begin to look for explanatory variables of sex-related differences in mathematics performance is in the cognitive area. It is within this area that the most important variable can be found; i.e., the amount of time spent studying mathematics. This variable and its impact have already been discussed.

Spatial Visualization

Another cognitive variable that may help explain sex-related differences in mathematics is spatial visualization—a particular subset of spatial skills. Spatial visualization involves visual imagery of objects, movement by the objects themselves or change in their properties. In other words, objects or their properties must be manipulated in one's "mind's eye"—or mentally. Even though the existence of many sex-related differences is currently being challenged, the evidence is still persuasive that in the American culture male superiority on tasks that require spatial visualization is evident beginning during adolescence (Fennema, 1975; Maccoby and Jacklin, 1974). However, even this difference appears to be moderating.

The relationship between mathematics and spatial visualization is logically evident. In mathematical terms spatial visualization requires that objects be (mentally) rotated, reflected and/or translated. These are important ideas in geometry. In fact, James and James (1968) in defining geometry as "the science that treats of the shape and size of things . . . the study of invariant properties of given elements under specified groups of transformation" (p. 162) are describing accurately most conditions which are met by items on spatial visualization tests.

Many mathematicians believe that all of mathematical thought involves geometrical ideas. The total discipline of mathematics can be defined as the language for describing those aspects of the world which can be stated in terms of "configurations" (Bronowski, 1947). Meserve (1973) believes that each person who makes extensive use of all areas of mathematics uses the modes of thought of geometry at every turn and that "even the most abstract geometrical thinking must retain some link, however attenuated, with spatial intuition" (p. 249). In the Russian literature, mathematics and spatial abilities are regarded as inseparable (Kabanova-

Meller, 1970). Therefore, if spatial visualization items are geometrical in character and if mathematical thought involves geometrical ideas, spatial visualization and mathematics are inseparably intertwined.

Not only are spatial visualization activities similar to ideas within the structure of mathematics, but spatial representations are being increasingly included in the teaching of mathematics. E.g., the Piagetian conservation tasks, which are becoming a part of many preschool programs, involve focusing on correct spatial attributes before quantity, length, and volume are conserved. Most concrete and pictorial representations of arithmetical, geometrical and algebraic ideas appear to be heavily reliant on spatial attributes. The number line, which is used extensively to represent whole numbers and operations on them, is a spatial representation. Commutativity of multiplication illustrated by turning an array 90 degrees, involves a direct spatial visualization skill. Many other examples could be cited.

Although the relation between the content of mathematics and spatial visualization skills appears logical, results from empirical studies which have explored the relationship are not consistent. Many factor analytic studies have explored this relationship and several authors have reviewed the literature. Some investigators have definitely concluded that spatial skills and learning of mathematics are not related. In 1967, Very concluded: "Research on spatial ability has failed to produce any significant correlation of (the spatial factor) with any facet of mathematics performance" (p. 172). Fruchter (1954) stated that "spatial ability is unrelated to academic performance with the possible exception of a few very specialized courses such as engineering drawing" (p. 2). Smith (1964) concluded that although

> ....there are several studies which indicate consistently that spatial ability is important in tests which are genuinely mathematical as distinct from those which involve purely mechanical or computational processes . . . the question whether the mathematical ability is dependent on the visual factor (or factors) has not been definitely answered. (pp. 127, 64).

Even in geometry where one would expect to find the strongest relationship, empirical findings do not indicate clearly that the two are related. Lim concluded in 1963 after a thorough review of relevant literature that the evidence for a relationship between geometric ability and spatial visualization was inconsistent and unreliable. Werdelin (1971) was not willing to conclude definitely that spatial visualization ability and geometry ability were related. However, he felt that "there is strong pedagogical reason to believe in a connection between the ability to visualize and geometric ability" (p. 39).

Other authors feel that data indicate a positive relationship. In 1951, Guilford, Green and Christensen concluded that spatial visualization ability helped in solving mathematics problems. French (1951, 1955) showed that successful achievement in mathematics depends to some extent on use of spatial visualization skills. In a more recent review, Aiken (1973) concluded that spatial-perceptual ability was one of the "most salient" mathematical factors extracted in various investigations. Obviously, the relationship between learning in mathematics and spatial ability is not clear and the need for more data is great.

Even less is known about the effect that differential spatial visualization has on the mathematics learning of females and males. Indication that the relationship between the learning of mathematics and spatial visualization is an important consideration, is the concurrent development of sex-related differences in favor of males in mathematics achievement and spatial visualization skills. No significant sex differences in either mathematics achievement or spatial visualization skills have been consistently reported in subjects 4-8 years old. Sex differences in performance on spatial visualization tasks become more pronounced between upper elementary years and the last year of high school (Maccoby and Jacklin, 1974). Sex differences in mathematical achievement that do exist also appear during this time span (Fennema, 1974). Perhaps less adequate spatial visualization skills may partially explain sex-related differences in achievement in mathematics.

However, the Fennema-Sherman study specifically investigated the relationship between mathematics achievement and spatial visualization skills. These data do not support the idea that spatial visualization is helpful in explaining sex-related differences in mathematics achievement. In this study of females and males (grades 6-12) enrolled in mathematics courses, few sex-related differences in either mathematics achievement or spatial visualization skills were found. The two were related ($\underline{r} = .5$) similarly for both sexes and spatial visualization appeared to influence both females and males equally to continue studying mathematics.

Affective Variables

The Confidence-Anxiety Dimension in Mathematics. One tends to do those things one feels confident to do and to avoid activities which arouse anxiety. This confidence-anxiety dimension, as it relates to mathematics learning, is one of the more important affective variables that helps explain sex-related differences in mathematics learning. The relationship of anxiety and mathematics learning has been explored by a variety of methodologies and with instruments purported to measure debilitative or facilitative anxiety in general and/or specific to mathematics. Callahan and Glennon (1975) conclude that "anxiety and mathematics are related. In general, high anxiety is associated with lower achievement in mathematics" (p. 82). Reports from NLSMA indicate that across grades 4-10 decreases in facilitating anxiety appeared, with females' scores decreasing more than males' scores. Debilitating anxieties increased for females during these grade levels (Crosswhite, 1975).

Confidence per se has not been given specific attention as it relates to mathematics except in the Fennema-Sherman study. However, self-concept, which appears to be defined in many scales as self confidence, has received much study. Leviton (1975) and Primavera et al (1974) reviewed the literature dealing with self-concept and both concluded that a positive relationship exists between academic achievement and self esteem. Brookover and Thomas (1964) offer evidence that self concept is not generalizable but related to specific academic areas. Callahan and Glennon conclude that there is a positive relationship between self-esteem and achievement in mathematics. Others have also recognized the importance of academic self concept in learning mathematics (Bachman, 1970; Fink, 1969).

Although both confidence and anxiety have been defined as separate traits, it appears in relation to mathematics, they are very similar. In the Fennema-Sherman study an attempt was made to measure both confidence and anxiety. A high rating on the confidence scale correlated highly ($r = .89$) with a low rating on the anxiety scale. While it may be possible to talk about the two independently, it doesn't appear to be useful.

The literature strongly supports the fact that there are sex-related differences in the confidence-anxiety dimension. It appears reasonable to believe that lesser confidence, or greater anxiety on the part of females is an important variable which helps explain sex-related differences in mathematics studying. Crandall et al. (1962) concluded that girls underestimate their own ability to solve mathematical problems. Others have concluded that females feel inadequate when faced with a variety of intellective, problem-solving activities (Kagan, 1964). Maccoby and Jacklin (1973) reported that "girls tend to underestimate their own intellectual abilities more than boys do" (p. 41).

In the Fennema-Sherman study, at each grade level from 6-12, boys were significantly more confident in their abilities to deal with mathematics than were girls. In most instances this happened when there were no significant sex-related differences in mathematics achievement. In addition, confidence in learning mathematics and achievement were more highly correlated than any other affective variable and achievement ($r = .40$). Confidence was almost as highly related to achievement as were the cognitive variables of verbal ability and spatial visualization.

Stereotyping Mathematics as a Male Domain. It is commonly accepted that mathematics is stereotyped as an activity more appropriate for males than for females. It has been believed that the sex typing of mathematics as male starts in the elementary school, becomes stronger during adolescent years and is solidified as a male domain by adult years. However, Stein and Smithells (1969) and Stein (1971) provide evidence that mathematics is not considered masculine by females and males until adolescent years and even during these years is not ranked as highly masculine as are spatial, mechanical and athletic tasks. Bobbe (1971) found that with fourth and sixth grade subjects, arithmetic was judged to be feminine by girls while boys judged it to be appropriate for both sexes. In the adult world, it is a fact that the use and creation of mathematics is predominantly a male domain. Stein and Smithells (1969) offered evidence that in 12th grade, females perceived this fact and were responding to the reality.

The Fennema-Sherman study indicated that females in grades 6-12 deny that mathematics is a male domain. While the males in the study did not strongly stereotype mathematics as a male domain, at each grade they stereotyped it at significantly higher levels than did females. This is an interesting and highly significant finding. The cross sex influence on all aspects of behavior is strong during adolescent years. Since males stereotype mathematics as a male domain, they undoubtedly communicate this belief in many subtle and not so subtle ways to females which influences females willingness to study mathematics. This has strong implications for the development of intervention programs designed to increase females' participation in mathematics.

Usefulness of Mathematics. A different kind of affective variable is belief in the personal usefulness of mathematics. Hilton and Berglünd (1974) and the Fennema-Sherman study provided data indicating that females to a lesser degree than males believe that mathematics is personally useful. However, the difference between female and male beliefs about the usefulness of mathematics was not as great in the Fennema-Sherman study as it was in the Hilton-Berglünd study. This may indicate that the beliefs of females are becoming more similar to males in this aspect.

Effectance Motivation in Mathematics. One variable, which has been hypothesized to show a sex-related difference, is effectance motivation. This motive can be "inferred specifically from behavior that shows a lasting formalization and that has characteristics of exploration and experimentation" (White, 1959, p. 323). It is closely related to problem-solving activity and is often called intrinsic motivation. This motivation would encourage learners to participate in mathematical activities at high cognitive levels. Some believe that females are not so involved in problem-solving activities as are males (Carey, 1958; Kagan, 1964). However, the Fennema-Sherman study found no sex-related difference in this variable at any grade level from 6-12. It appears that belief that females are not as intrinsically motivated as males in mathematics is merely a myth.

Educational Variables

There are sex-related differences in the final outcome of mathematical education due in large part to females' reluctance—if not refusal—to elect to study mathematics. Some intervention is essential at the present time to ensure equity in mathematical education for both sexes. However, before effective intervention can be planned, more information is needed about critical school variables which are amenable to change and important in the educational process.

Teachers. Teachers are the most important educational influence on students' learning of mathematics. From kindergarten to high school, learners spend thousands of hours in direct contact with teachers. While other educational agents may have influence on educational decisions, it is the day-by-day contact with teachers which is the main influence of the formal educational institution. Part of the teachers' influence is in the learners' development of sex role standards. These sex role standards include definitions of acceptable achievement in the

20

various subject areas. It is believed that this influence by teachers is exerted through differential treatment of the sexes as well as expectations of sex-related differences in achievement.

Schonborn (1975) concluded that many studies have indicated teachers treat female and male students differently. In general, males appear to be more salient in the teachers' frame of reference. Teachers' interaction with males is greater than it is with females in both blame and praise contacts. Teachers also reinforce in both females and males sexually stereotypic behavior deemed appropriate for their sex (Sears and Feldman, 1966). Brophy, Good and their colleagues have been the major investigators of teacher treatment of females and males with their main interest being teachers' treatment of males. In several studies they have concluded that girls and boys receive equal treatment. However, the data from one of their major studies shows that while the sex of the teacher was unimportant, high-achieving high school boys received significantly more attention in mathematics class than any other group (Good, Sikes and Brophy, 1973). Another study involving first grade reading replicated this trend at nonsignificant levels (Good and Brophy, 1971). Their conclusion from these studies is that teacher bias was not evident. One must question why no conclusion was reached about inequitable treatment of high-achieving females.

The investigation of the relationship between teacher behavior and sex-related difference in mathematics appears to be crucial to understanding why females do not participate at higher levels in mathematics. In particular information in these areas would be helpful: 1) What are the effects of differential teacher treatment and expectations on achievement and election of mathematics courses? 2) Do teachers differentially reinforce males and females for specific kinds of mathematical and/or sexually stereotypic activities? Are males being reinforced more for problem-solving activities while females are reinforced for computational activities? 3) What is the effect of sex of teacher on mathematical achievement of boys and girls? While O'Brien (1975) reports no sex of teacher effect, Good, Sikes and Brophy (1973) and Shinedling and Pedersen (1970) report that male students do best in quantitative scores when taught by male teachers.

School Organization. There is some evidence and much belief that schools do influence sexual stereotypes. Minuchin (1971) concluded that children who attended schools categorized as traditional or modern differed in their sex-typed reactions. The interaction of the sexes was different in those schools, also. In the most traditional school boys became leaders in problem solving while girls became followers. This was not so in the less traditional schools. The sex role behavior of children attending traditional schools was more rigid than children attending liberal schools.

Some schools are remarkably more effective in persuading females to attempt high achievement in mathematics. Casserly (1975) identified 13 high schools which had an unusually high percentage of females in advanced placement mathematics and science classes. She concluded that the schools had identified these girls as early as fourth grade and the school teachers and peers were supportive of high achievement by the females. Rowell (1971) pursued the same type of investigation in attempting to identify schools and their characteristics which produced high-achieving females in science. Studies identifying and describing those schools which are particularly successful in encouraging females to enroll in mathematics beyond minimal requirements should be done.

Many are advocating that female only classes will result in equity in mathematical education. The argument for this type of school organization goes something like this. Because peer pressure against female competitiveness is too strong a force, females will not compete against males in mixed sex classrooms. Female leadership (in problem solving in this case) is only able to emerge when competition with males is eliminated. Teachers will not have different sex-related expectations and behaviors if only one sex is present. Single sex classrooms appear to provide a simple solution to a complex problem. However, the weight of evidence found does not support this type of grouping. Conway (1973) convincingly argues that throughout history separate education for the sexes has resulted in inferior education for females. Keeves (1973), after a careful and thorough review of mathematics and science education in ten countries, concluded that the "extent to which a community provides for education in single sex schools would appear to indicate the extent to which it sees its boys and girls requiring different preparation for different societal roles" (p. 62). He argues that "insofar as a community has different expectations for different groups of its members and proceeds to mold its future members through different organizations, then it fails to provide equal opportunities for individual development" (p. 52). In an unreported study comparing attitudes of 10th grade females who had spent most of their educational lives in single or mixed sex classrooms, females from the mixed sex classrooms exhibited significantly more positive attitudes toward mathematics (Fennema-Meyer, 1976).

Before single sex classrooms are embraced as a panacea for females' educational equity, careful examination must be done concerning long-term effectiveness of such programs. In reality, this may be a partially non-researchable problem. No one can foresee the implications for females 50 years from the present time of being isolated in their mathematical training. Because of what has happened to females as well as blacks over the last century, single sex classroom school organization must be approached with caution.

Conclusions

What then, can be said that is known about sex-related differences in mathematics, and factors related to such differences? Certainly, when both females and males study the same amount of mathematics, differences in learning mathematics are minimal and perhaps decreasing. Many fewer females elect to study mathematics and therein lies the problem. Factors which appear to contribute to this nonelection are females' lesser confidence in learning mathematics and belief that mathematics is not useful to them and males' belief that mathematics is a male domain. In addition, differential teacher treatment of males and females is important.

There is nothing inherent (Sherman, 1976) which keeps females from learning mathematics at the same level as do males. Intervention programs can and must be designed and implemented within schools which will increase females' participation in mathematics. Such programs must include male students, female students and their teachers. Only when such intervention programs become effective can true equity in mathematics education be accomplished.

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ARE MATHEMATICS PROBLEMS A PROBLEM FOR WOMEN AND GIRLS?

Ann K. Schonberger

International Women's Year in 1975 focused attention on improving the status of women. In this country, women's demands for equality have led to identification of a number of target areas in which women's roles have been limited in the past: primary among these are the economic and intellectual activities of the nation. Educators in general and mathematics educators in particular are now beginning to address themselves to broadening and improving the economic and intellectual position of women.

In the economic sphere men far outnumber women in occupations requiring a high level of mathematical competence. Various causes for this imbalance have been hypothesized: sex bias in career counseling, discrimination in admission to specialized schools, and differences in sex-role socialization. In addition, differences in mathematical ability are often suggested as reasons (Carnegie Commission, 1973). Some have said that while girls may be better at computation, boys excel at mathematical reasoning (Glennon and Callahan, 1968; Jarvis, 1964; Maccoby, 1966). If this is true, mathematical reasoning could be the "critical filter" (Sells, 1973) in the scientific and technical job market, since in those occupations the application of mathematics to problems is valued more highly than computing skill. Furthermore, not all economic activity is job related. As women become more independent in buying and maintaining homes, cars, and other products, they need to be able to solve mathematics problems of a practical nature. The traditional female duties of providing food and clothing continue to pose mathematical problems to be solved.

In the intellectual sphere the importance of problem solving is not to be underestimated. To quote George Polya (1962, p. v.), "Solving problems is the specific achievement of intelligence and intelligence is the specific gift of mankind [sic]: solving problems can be regarded as the most characteristically human activity." Furthermore, solving mathematics problems has been regarded as the essence of human problem solving; psychologists often use mathematics problems in their general problem solving research (Berry, 1958, 1959; Carey, 1955; Milton, 1957, 1958; Simon and Newell, 1971; Sweeney, 1953). Mathematics educators themselves stress the value of teaching problem solving.

> Our instruction serves to develop the capacity of the human mind for the observation, selection, generalization, abstraction, and construction of models for use in solving problems in other disciplines. Unless the study of mathematics can operate to clarify and solve human problems, it has indeed only narrow value.
>
> Fehr, 1974, p. 27

Paper presented at the annual meeting of the National Council of Teachers of Mathematics, San Diego, April, 1978.

Having recognized the importance of being able to solve mathematical problems, one should return to this question: Is it true that males excel at mathematical problem solving? Before examining the research literature on that question, it would be well to adopt a precise definition of a mathematical problem.

> A mathematical problem is a statement which meets three conditions:
>
> 1) the statement presents information and an objective based on that information;
>
> 2) the objective or answer can be found by translation of the information into mathematical terms and/or application of rules from mathematical areas such as arithmetic, algebra, logic, reasoning, geometry, number theory or topology; and
>
> 3) the individual attempting to answer the question or attain the objective does not possess a memorized answer or an immediate procedure.
>
> Zalewski, 1974, pp. 4-5

The third part of the definition serves to set real problems apart from routine applications or exercises. To qualify for inclusion in this review a study must have used test items intended by their designers to measure mathematical behavior other than computation and items which seemed to this author to satisfy the definition.

A second question guided the organization of this review. If males as a group are better at solving mathematical problems than females, when does this superiority develop? In order to investigate that question the studies were grouped as follows: elementary; Grades 7 and 8; Grades 9 through 12; college and adult. In fact, since most mathematics tests for primary grades are computational, the elementary studies reported are from Grades 4 through 6. Whenever possible two additional refinements of the original question were investigated. Do the results depend on whether the content of the problems is algebra, geometry, or real-life situations? Are the results different for different ability groups?

Several of the studies which contributed most to this section of the review are longitudinal and a word needs to be said about their methodology. One is the National Longitudinal Study of Mathematical Abilities (NLSMA), probably the most intensive and extensive study in this area. Three different groups were tested: one in Grades 4 through 8, another in Grades 7 through 11, and the third in Grades 10 through 12. A content (number systems, geometry, algebra) by level of behavior (computation, comprehension, application, analysis) matrix was used to categorize the mathematics scales. Both application and analysis scales were included in this review, although the definition of analysis items seems closer to the definition of a mathematical problem. The study, designed to compare certain textbook series, involved primarily college-capable students. Another factor which should be considered in evaluating the results is that the sex-related differences reported were those remaining after removal of the variance due to verbal IQ, nonverbal IQ, and mathematics achievement. A second longitudinal study (Milton and Berglund, 1974), whose results are reviewed here, measured the same students in Grades 5, 7, 9, and 11 using the Sequential Test of Educational Progress-Mathematics (STEP-Math) which the authors regarded as a measure of the ability to apply skills to problem solving. The sample was divided into an academic group and a nonacademic group according to what program they eventually pursued in high school, and results were analyzed separately by group.

In the NLSMA study of Grades 4 through 6 boys excelled on two out of three applications scales, both concerned with number systems, and on the only analysis scale, a geometry scale (Carry and Weaver, 1969). Hilton and Berglünd found no significant differences in either fifth-grade group on STEP-Math. In a study using fifth-grade subjects Harris and Harris (1973) found no sex-related differences on either of the two cognitive abilities tests containing mathematical problems. Similarly no differences between boys' and girls' performances on an arithmetic reasoning test were found by Parsley, Powell, O'Connor, and Deutsch (1963). A second study (1964) by those authors indicated better performance by males on 12 subgroups and by females in 7 subgroups out of a total of 75 comparisons. In a study of sixth-grade students Jarvis (1964) found that boys of all ability levels surpassed girls in arithmetic reasoning. Clearly, although some differences have begun to appear in upper elementary school, the results are mixed.

Sex-related differences were more apparent in the studies reviewed using seventh- and eighth-grade students. Hilton and Berglünd reported a difference in favor of boys on STEP-Math in the academic group. The NLSMA also gave STEP-Math to one group in seventh grade, categorizing it as an application test, and found boys' performance to be superior (McLeod and Kilpatrick, 1969). Sex-related differences in favor of boys were also found on all but one of the analysis scales and on the one application scale designed by NLSMA (Carry 1970; McLeod and Kilpatrick, 1969). The content of the scales on which differences were found was number systems and geometry; the scale on which none were found was an algebra scale. In a study of problem-solving styles in high-ability, eighth-grade subjects Kilpatrick (1967) found that although scores for boys and girls were about the same, girls used significantly more deduction and more equations. In the National Assessment of Educational Progress (NAEP) consumer math skills were measured by a test of problems given to 13-year-olds, 17-year-olds, and young adults ages 26 to 35. In the youngest group the boys' median was 1-1/2 percent above the median of the total group and the girls' median was 1-1/2 percent below (Ahmann, 1975).

With the exception of the NAEP all the studies discussed in this section in which sex-related differences were observed were conducted with students of above-average ability. There is another indication that overall superiority of boys in mathematical problem solving in Grades 7 and 8 may be due to superior performance of boys of high ability. In a study of mathematical precocity Stanley, Keating, and Fox (1974) found that in a sample of seventh- and eighth-grade students who volunteered for screening with the Scholastic Aptitude Test-Mathematics (SAT-M) boys far outperformed girls and the discrepancies increased with age. These results ought to be tempered by the observations of Donlon (1973) who studied the sex-related differences in performance item by item on both the verbal and mathematical sections of the SAT given in May 1964. Of the 60 items on the SAT-M, 17 were found to refer to real world things. There were no female agents in these 17 items and, according to Donlon, there seemed to be a masculine tenor to the items. This was also the type of item on which the sex-related difference in performance in favor of males was the greatest.

Surveying the studies of high school students required additional caution because required mathematics courses are more often tracked and mathematics becomes elective in the upper grades. Good examples of this lack of control for number or type of mathematics courses taken are the Project Talent Survey (Flanagan, et al., 1964) and the NAEP (Ahmann, 1975) both of which found sex-related differences in favor of males. In all the other high school studies reviewed here the students were in the same mathematics class or track when tested.

Information on sex-related differences in the NLSMA were reported only for the college-preparatory group. At the applications level boys in Grades 9 through 11 excelled girls on 5 out of 12 geometry scales and the one algebra scale. At the analysis level their performance was superior on half the algebra and number systems scales; on the geometry analysis scales boys excelled on 6 of the 8 and girls on 2. (Kilpatrick and McLeod, 1971a, 1971b; McLeod and Kilpatrick, 1971; Wilson, 1972a, 1972b.)

The impression of overwhelming evidence of male superiority on NLSMA mathematical problems solving tests should be tempered by limitations of the study due to sampling, statistical analysis, and problem content of the tests. The sampling restriction to upper-ability students was more severe in the high school data than in the junior high data. The statistical removal of variance due to verbal and non-verbal IQ and mathematics achievement may have left only a small fraction of the variance. Application of the w^2 statistic (Hayes, 1973) to three of the analysis scales given in Grade 11 showed that on each less than 1 percent of the variance was due to sex. With respect to problem content, sex-related differences in performance on the analysis scales appeared most numerous in the area of geometry, which may be related to the reported male advantage on spatial abilities (Bennett, Seashore and Wesman, 1973; Maccoby and Jacklin, 1974). One of the two geometry scales on which girls excelled was Structure of Proof, which appeared to require verbal rather than spatial skills. Finally, the content of the number systems problems for Grades 4 through 11 should be considered. Among these were all the problems about people. In virtually all cases in which sex of a person was specified, the person was male.

Evidence of the importance of sampling, statistical analysis, and problem content was found in other high school studies. Differences dependent on sampling of ability levels were evident in the Hilton and Berglünd study where boys from the academic group scored significantly higher than girls in Grades 9 and 11, whereas in the nonacademic group boys scored higher than girls only in eleventh grade. Statistical techniques were used by Sheehan (1968) in a study of problem solving in ninth-grade algebra to change a slight (but nonsignificant) advantage of girls into a significant difference in favor of boys by removal of the variance due to algebra aptitude and previous mathematics achievement and knowledge of algebra. In his high-ability, Swedish, high school students Werdelin (1961) found sex-related differences limited to two tests of geometrical problems. The importance of problem content was also demonstrated by Leder's (1974) study using mathematically parallel pairs of problems with stereotypically male and female settings. Tenth grade boys and girls both preferred the problems appropriate to their traditional sex-roles.

Table 1

Sex-Related Differences in Analysis Scales
National Longitudinal Study of Mathematical Abilities

Grade	Number Systems			Geometry			Algebra		
	Total	Boys[a]	Girls[b]	Total	Boys	Girls	Total	Boys	Girls
4									
5				1	1	0			
6									
7	2	2	0	2	2	0			
8	3	3	0	4	4	0	1	0	0
9				1	1	0	1	1	0
10				6	4	2			
11	2	1	0	2	2	0	2	1	0

[a] Number of scales on which boys' performance was significantly better.

[b] Number of scales on which girls' performance was significantly better.

Studies of college students and adults are even more open to criticism for lack of control for previous exposure to mathematics. Very's (1967) study and the NAEP, both of which found males to be better problem solvers, can be criticized on this point. However, there is a significant group of interrelated studies of problem solving in college students in which previous mathematics training was controlled. After Sweeney's 1953 study in which sex-related differences in addition to those due to intellectual factors were found, the others investigated various other noncognitive sources of the difference. Carey (1955) found attitude toward problem solving to be a significant factor in males' better performance on the problem test. Moreover, following a treatment designed to improve attitude, women's problem-solving performance improved significantly, whereas men's did not. Milton (1957, 1958) and Berry (1958, 1959) investigated the relationship between the Terman-Miles masculinity-femininity index and mathematics problems similar to those used by Sweeney and Carey. In only one of the four studies was the correlation significant after the removal of the effects due to verbal and quantitative factors. In the 1959 study Berry used a number of other noncognitive measures and found that the only ones contributing to the remaining problem solving variance were two tests of spatial ability and Carey's attitude test—and this only for males. Milton also investigated the effects of problem content and found men superior at solving "masculine" but not "feminine" problems.

Table 2
Sex-Related Differences in Applications Scales
National Longitudinal Study of Mathematical Abilities

Grade	Number Systems			Geometry			Algebra		
	Total	Boys[a]	Girls[b]	Total	Boys	Girls	Total	Boys	Girls
4	1	0	0						
5	1	1	0						
6	1	1	0						
7	1	1	0						
8	1	1	0						
9									
10				6	4	0	1	1	0
11				6	1	0			

[a] Number of scales on which boys' performance was significantly better.

[b] Number of scales on which girls' performance was significantly better.

All of the studies reported thus far were done before 1975 and, with the exception of the college studies just described, were designed to study other areas besides sex-related differences in problem solving. Summarizing them is difficult. The sex-related differences may have been small but they did seem to exist, even after controlling for mathematics background. Differences appeared in early adolescence and may have increased with age until maturity. The pre-1975 studies indicate that sex-related differences may be found only with subjects of above-average ability and on problems whose content is spatial or sex-biased. Differences on sex-biased problems suggest that social pressures prevent females from solving problems as successfully as males. Since 1975 research has been done on problem solving that either concentrated on the areas identified above or at least showed an awareness of the issues involved with using sex as a variable.

Sex bias in item content of problem solving tests is the area in which results of recent studies have been most definitive. There are several ways of assessing such bias and eliminating it. Faggen-Steckler, McCarthy, and Tittle (1974) developed a technique for measuring sex bias in tests based on the ratio of male nouns and pronouns. They calculated the ratios two ways—both including and excluding such generic nouns as chairman, mailman and mankind. Using this procedure they examined eight standardized achievement test batteries among those most frequently used in public schools; among these batteries were the Sequential Tests of Educational Progress on which sex-related differences had been observed in the NLSMA and the Hilton-Berglünd study discussed earlier. All eight batteries were sex biased in favor of males in varying degrees. It should be noted, however, that in this study entire test batteries were evaluated, not the mathematics tests in particular.

Another technique for trying to eliminate sex bias in problem solving tests is to treat each item as a unit. A study of seventh-grade students by the author (Schonberger, 1976) used a test constructed using the following technique. Whenever possible the problem was made neuter. For example, "Six girls belong to a basketball team" became "Six students belong to a basketball team." Where this was not possible, names and pronouns were changed so that there were equal numbers of male-acted and female-acted problems in each subtest. There were no significant sex-related differences between girls' and boys' performances on two of the three subtests or on the total test. The better performance by males on the third subtest was due primarily to one item which involved comparing shooting averages in a ball game, a task which may have been more familiar to boys than to girls. This one difference in favor of males was not more pronounced in the upper ability group.

The Romberg-Wearne Problem Solving Test (Wearne, 1976) which has concepts, applications, and problem solving subtests and has been used in several recent studies was also balanced by equating numbers of male-centered and female-centered items. Meyer (1976) used this test in a study of cognitive abilities and problem solving abilities of fourth-grade students; she found no significant sex-related differences. Fennema and Sherman also used the Romberg-Wearne test in their middle-school study (1978) and found sex-related differences in only one of the four areas of the city used in the study. In summary, those recent studies of children in Grades 4 through 8 suggest that when an effort is made to eliminate sex bias from the tests, fewer sex-related differences appear.

The elementary and middle school studies just reviewed focused primarily on the actors in the problems as a source of sex bias. However, the author's study showed that the content of items may also be a source of bias if the topic, such as sports, is more interesting or familiar to one sex than the other. McCarthy's (1976) study of problem solving using students in Grades 10 through 12 dealt with this aspect of bias. A group of students categorized each of a large battery of problems similar to those used on the SAT-M as masculine, feminine, or neutral. Another group of students were then tested with 26 of each type of item. Using a technique common in test construction, McCarthy computed the point-biserial coefficient for each problem for the males, the females, and the total group to identify the 26 items which best discriminated between high and low scorers in each group. Of the 26 best discriminators for girls, 14 had been categorized as feminine items, 2 as masculine, and 10 as neutral; of the boys best discriminators

11 were masculine, 6 were feminine and 9 were neutral. The distribution for the total group best discriminators was close to that for the boys: 10 masculine, 4 feminine and 12 neutral.

Twelve items appeared in all three groups of best discriminators, but performance by males and females differed significantly on the three tests.

Table 3

Comparison of Means in McCarthy's Study
(McCarthy, 1976, p. 55)

	Male Means	S.D.	Female Means	S.D.
Total Group Discriminators	17.90	5.38	14.36	6.34
Male Group Discriminators	16.02	5.66	12.93	5.64
Female Group Discriminators	13.98	5.20	15.61	6.49

Females did best on the female group discriminators and least well on the male group discriminators. Male performance on female discriminators was better than female performance on male discriminators, and male performance on male discriminators was better than female performance on female discriminators, but the differences were certainly smaller than the male-female differences on the total group discriminators. This is important because under common test construction procedures the total group discriminators would have been used as the problem solving test.

So far the discussion of recent studies has focused on the sex of actors in the problem or on sex-role stereotyped interests such as sewing and cooking for females and business or sports for males. These types of problems were categorized as number systems problems by the MLSMA, which found them generally easier for boys as a group than for girls. The other type of problems on which males outperformed females in some of the earlier research was geometry problems. Two questions have guided recent research in this area.

1. Do males still outperform females on mathematical problems with spatial or geometric content?

2. If so, is the male advantage in problem solving related to male advantage on tests of visual spatial ability?

The author's (1976) study investigated the first question by constructing a problem solving test with three types of problems.

A. Problems in which the stimulus (presentation of the problem) is partly pictorial or which require spatial or geometric skills or knowledge for solution.

B. Problems with a completely verbal stimulus in which spatial skills (such as visualizing the situation or drawing a diagram) may be useful but are not necessary for solution.

C. Problems which appear to have no spatial content.

In the seventh-grade sample used in this study there were no significant sex-related differences in performance on Type A or Type B problems. The significant difference in favor of males on the Type C problems was due to sex-bias of one item dealing with sports as discussed previously.

According to recent studies by Meyer (1976) and by Fennema and Sherman (1978) as well as by the author, superiority in problem solving and spatial ability are not necessarily related. In Meyer's study of fourth grade students there was a sex-related difference in favor of males on the Space Relations test from the Primary Mental Abilities battery but no difference on the Romberg-Wearne Problem Solving Test. In the Fennema-Sherman middle school study there were no significant sex-related differences in the Space Relations Test of the Differential Aptitude Test, even in the area of the city in which differences in favor of males had been found on the Romberg-Wearne test. In the author's study five spatial tests were used including the DAT Space Relations test and a form board test, the type used by Meyer. Only on the form board test did boys excel, and this test was the least closely related to the problem solving measures of any of the five spatial tests. The question just asked about geometric problems and spatial ability have not been investigated with high school students in recent studies. Given the NLSMA results and similar findings by Donlon (1973) with respect to the SAT-M, it seems that further research at the high school level would be useful to see if females still perform less well on spatial or geometric problems.

If such research does find that females perform less well on certain types of questions, it raises important issues in the area of test construction. As Donlon pointed out, the male "advantage" of 40 points on the SAT-M data he studied could grow to 60 points if only real life subject matter items were used or diminish to about 20 points if only algebra-type items were used. If there are certain types of items on which males do better than females, is it justifiable to eliminate them? If there are certain types of items on which females do better, is it fair to increase their numbers? These are difficult questions to answer, and are ultimately related to the content validity of the test. While the author sees no difficulty in eliminating sports problems dealing with batting averages, the elimination of all spatial or geometric content from a test of mathematical problems seems illegitimate. Whether or not different item formats weight test performance in favor of males or females should also be investigated.

McCarthy commented on another test construction issue, the possible sex bias in the point-biserial technique for selecting items from a pool which best discriminate between high and low scorers. She argued that if there are more males than females among the high scorers and more females than males among the low scorers, then the point-biserial technique selects items on which males do better than females. Indeed with her data the best discriminators for the total group were 10 masculine, 4 feminine and 12 neutral items. She experimented with different ways of eliminating this bias such as selecting the nine best items in each of the masculine and feminine categories and the eight best neutral items. She found the least difference in male and female performance by using only neutral items but decided that the restriction in content was unacceptable.

While most educators would agree that separate tests for males and females are not feasible, the technique of balancing a test with equal numbers of "masculine" and "feminine" items also can be questioned. Is it legitimate to perpetuate sex-role stereotypes by using problems in which girls cook, sew and lose weight while boys build, fish and buy cars? The author thinks not. However, the following test construction strategy offers a possible solution. As many items should be made neutral as possible without lowering the content validity of the test. The remaining set of items should be balanced with respect to male and female actors as well as with respect to stereotypically masculine and feminine content. However, there should be females with hammers and saws as well as males with pots and pans, and female cab drivers as well as male teachers.

Constructing sexually fair tests may not be sufficient to eliminate all the sex-related differences in problem solving performance. There were indications in the earlier research (Carey, 1955) that females' attitudes toward problem solving were inhibiting their performance. In a study by McMahon (McCarthy, 1976) students in Grades 6 and 10 and college freshmen were asked to predict their own success on scrambled word problems and arithmetic problems after being assured of their familiarity with the problems. Although there were no differences in males' and females' expectations of success on the word problems, females at all three ages significantly underpredicted their success on the arithmetic problems.

More recent research indicates that social expectations and pressures are probably still affecting women's performance. In Fennema's and Sherman's study the one area of the city in which males outperformed females on the problem solving test was the one in which sex-related differences were observed on six of the eight attitude measures. Hall (1976) studied high-ability high school students solving problems in groups of four and offered this observation.

> The constituency of a team appeared to affect its performance. For example, sexually mixed teams appeared to lose input from female members because of male dominance of conversation or assignment of a female to the role of recorder (secretary).
>
> Hall, 1976, p. 55

However, a more positive note is that such social pressure may not be uniform across ethnic groups. Schratz (1976) observed a trend for white adolescence males to outperform white females in mathematical reasoning, but the trend was reversed for black adolescents.

In summary, the results of recent studies of sex-related differences in problem solving are both encouraging and useful. Efforts to eliminate sex bias in tests has eliminated or reduced the difference between male and female performances in problem solving. Although one study of seventh-grade students indicated that geometric or spatial content was no longer a stumbling block for females, studies of high school students in this area would be useful. The remaining differences are probably involved with social attitudes toward problem solving as a male activity. It is in this area of changing the attitudes of students of both sexes that teachers will have to be most tactful, most inventive, and also most reflective. The problem with problems has not disappeared entirely, but the changes that have occurred suggest that further change is possible. As mathematics educators we should commit ourselves to doing our part to enable women to participate fully in our mathematical society.

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WHY SUSIE CAN'T—OR DOESN'T WANT TO—ADD

Grace M. Burton

> Today, more than ever before, the study and appreciation of mathematics are vital to the intellectual development and to the scientific, industrial, technological, and social progress of society. It is essential that teachers, counselors, supervisors, educational administrators, parents and the general public work together to provide the best mathematics education possible for all students, regardless of sex...
>
> NCTM Statement on Counseling, 1976

> Many women in our present culture value mathematical ignorance as if it were a social grace.
>
> Lynn Osen, Women in Mathematics, 1974

"Add" in the title of this presentation is used in a generic sense—it includes the whole realm of mathematics—computational prowess, spatial skill, and reasoning ability. There is little doubt that women are at least the equals of men on measures of general intelligence, and that in elementary school, girls perform at least as well as boys in all subjects, including mathematics (Maccoby and Jacklin, 1974; Callahan and Glennon, 1975). At the junior high level, there are about as many girls as boys in Algebra I classes. By the time senior year rolls around, the ratio of males to females in advanced mathematics courses approaches 3 to 1 (Fennema and Sherman, 1976). Data from Berkeley indicates that while 57 percent of the freshmen males had taken four years of high school mathematics, only 8 percent of the females had (Sells, 1973). By the third semester of college, the male/female ratio in mathematics courses is 6 to 1 (Fennema and Sherman, 1976). In 1975, although a heartening 41 percent of the bachelor's degrees in mathematics were awarded to women, female students earned only a meager 2.2 percent of the bachelor's degrees in engineering (Westervelt, 1975; Eiden, 1976).

The message that Susie can't, or shouldn't, or doesn't need to, excel in mathematical endeavors is well-entrenched in American folklore and in the reality of the classroom. Other papers in this NCTM women and mathematics strand have focused on the research pertinent to this issue. I would like to approach the topic in a slightly different vein, considering the ways in which personal and social value structures contribute to the genesis of this problem, then suggesting a framework in which change can take place, and finally delineating a set of action steps which have proven beneficial in school districts and/or college classrooms in helping Susies become aware that achievement in mathematics is a proper goal for any person, regardless of sex.

When one sets out to solve any significant problem, three separate components of the solution must be considered. One must have knowledge of the problem and its ramifications, one must get in touch with his or her feelings about the issues under consideration, and one must act in such a way as to bring about a solution (see Figure 1). To engage in research, to read research completed by others, to exchange ideas with colleagues and to formulate models, all these are important in establishing a knowledge base; they contribute to the component labeled Thinking.

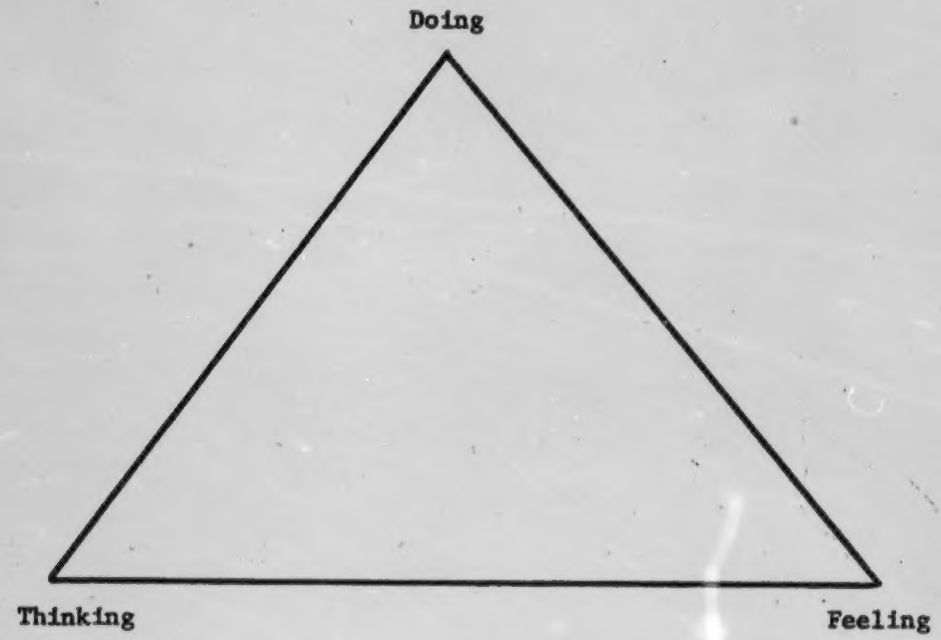

Fig. 1. Components for producing change.

To become aware of one's "gut level" reaction to a situation, to vent one's anger or disgust, to form a firm resolve to do something about a distressing situation, all these responses are important to the Feeling dimension. But neither of these essential considerations will bring about change. Change is accomplished only by Doing. Library or laboratory investigations may be intriguing and sessions of "Poor me, I've been done ill by," may be a pleasant way to pass the time, but if the status quo is to be changed, a commitment to action and action itself, is essential.

This does not imply that thinking and feeling about a problem are not important. Both have a role to play in the change process. A simple exercise can help clarify these dimensions. Draw a line vertically on a long sheet of paper and head one column "I think," the other "I feel." Then list in each column appropriate entries concerning the situation under consideration. For example, in one column you might list what you know about why women do not do well in mathematics at the post secondary level. In the other column you might write what you feel about this issue.

(This instructive exercise can be done in pairs also. Two people can sit facing each other. One of them speaks for two minutes on what he or she *feels* about the subject; the other listens. Then the pair reverses roles. The procedure is repeated with each person in turn speaking on what each *thinks* about it.)

It is convenient, but misleading, to assume that simply presenting objective, rational reasons for change will bring about a desired alteration in behavior of an attitude. The presentation of information may impact the cognitive domain, the "thinking" part of a person; it probably leaves untouched the affective dimensions of a person, the "feeling" part. Separating these two areas, then, can provide an individual with insight into why he or she behaves in certain ways. If change is desired in the cognitive domain, information may help; if change is desired in the affective domain, it will probably not. "Gut level" responses cannot be intellectualized away. Much of what has been traditionally accepted as truth, however, has been a blend of fact and feeling, or of opinion masquerading as fact.

There is available an enormous range of pontification from a wide variety of people concerning the suitability of intellectual endeavors for females. Much of it confuses what is *known* with what is *felt*. It may be interesting to examine the roots of our present socialization patterns in the light of some pronouncements from the past:

> The education of women should always be relative to that of men. To please, to be useful to us, to make us love and esteem them, to educate us when young, to take care of us when grown up, to advise, to console us, to render our lives easy and agreeable... Even if she possessed real abilities, it would only debase her to display them.
>
> J.J. Rousseau, *Emile*, 1762

> The glory of man is knowledge, but the glory of women is to renounce knowledge.
>
> Chinese Proverb

> The woman is naturally made for two things: household and domestic tasks and the exercise of pure love and devotion... On the contrary, man has compartments, sectors, and pigeon holes in his mind; he likes things to be separate and each in its order. And, although there are sublime exceptions, he is geared for mathematics, for arrangement, for the arts and sciences; he is as ill-at-ease at housekeeping as mysticism.
>
> Jean Guitton, *Feminine Fulfillment*, 1965

> When expressing your viewpoint use words that indicate insight such as "I feel." Avoid the words "I think," or "I know."
>
> Helen B. Andelin, *Fascinating Womanhood*, 1965

Since women are deficient in reason but abundant in emotion, they ought no longer to be treated as rational, nor receive any mental education... Among women, we use language implying the utmost deference for their sex; and they fully believe that the Chief Circle Himself is not more devoutly adored by us than they are; but behind their backs they are both regarded and spoken of -- by all except the very young -- as being little better than "mindless organisms."

Edwin A. Abbot, Flatland, 1884

Men have broad shoulders and narrow hips and accordingly they possess intelligence. Women ought to stay at home; the way they were created indicates this, for they have broad hips and wide fundament to sit upon.

Martin Luther, Table Talk, 1569

When a woman inclines to learning there is usually something wrong with her sex apparatus.

F.W. Nietzche, Beyond Good and Evil, 1886

The sight of the female form tells us that woman is not destined for great work, either intellectual or physical. (Woman is) a kind of intermediate step between the child and the man, who is a human being in the real sense.... Women exist solely for the propagation of the race.

Schopenhauer, "On Women," 1851

In the East, women religiously conceal that they have faces; in the West, that they have legs. In both cases they make it evident that they have but little brain.

Thoreau, Journal, 1852

A girl should not be too intelligent or too good or too highly differentiated in any direction. Like a ready-made garment she should be designed to fit the average man.

Emily Jane Putnam, The Lady, 1910

The female's chromosones, hormones and physical structure make it natural for her to respond, to follow and to submit.

Richard W. DeHann, Male, Female, and Unisex, 1977

Woman has "come down from the trees" more slowly than man has.

Theo Lang, The Difference Between a Man and a Woman, 1971

> She told me it was a girl, and I turned my head away and wept. "All right," I said, "I'm glad it's a girl and I hope she'll be a fool—that is the best thing a girl can be in this world, a beautiful little fool."
>
> Daisy in The Great Gatsby
> (By F. Scott Fitzgerald), 1925

There have long been those who have asserted that men are superior to women in intellectual areas simply because they were men; that is, that maleness itself implied intellectual superiority. By many, the two sexes were believed divinely determined to be as different with respect to psychological traits as they were to biological traits. The lack of coeducation, coupled with the lack of standardized testing procedures, hampered research on this question until the present century.

There are, of course, some established differences in male and female development. Males mature more slowly from birth onward, and, in fact, are more vulnerable to environmental insult during the prenatal period and throughout life (Maccoby and Jacklin, 1974). While few believe that mathematical ability is a province of the male sex, some believe there may be biological reasons for male superiority in one facet of mathematical ability—that of spatial visualization. Spatial visualization includes the ability to rotate objects in the mind and to orient oneself or other objects in space. While these abilities may be of importance in solving problems in certain areas of mathematics (geometry, for example), there are many other areas of mathematics in which they are not. Even in the former areas, lack of spatial ability is often compensated for by verbal ability. Two people who solve a geometric problem, for instance, may utilize different strategies—one abstracting relationships through a visual mode, the other "talking the problem through."

Currently there are three biological reasons advanced to explain spatial ability differences between the sexes. One postulates the existence of a recessive gene on the x-chromosone which enhances spatial ability in about half the male and one quarter of the female population. A second biological argument asserts that the male sex hormones play a role in enhancing spatial ability. A third theory concerns brain lateralization. The data on which this theory is based is derived primarily from medical research on patients with brain lesions and strokes and from animal research. Evidence is presented that the right hemisphere, usually considered the site of spatial processing, develops differently in males and in females. At present, support for these biological theories is not conclusive.

Explanations that mathematical differences between the sexes is biological in nature have been offered in present times to explain only one facet of mathematical ability. Historically, biological explanations were not as parsimonious. Until the turn of the century, many people believed that women were innately incapable of rigorous study, and that if such study were attempted, consequences to the woman and to her future offspring would be serious. Women were thought to have nerves which were smaller and more delicate than those of men, that to allow a young woman to study would cause her to "loose her bloom." Serious intellectual activity during adolescence, it was believed, would cause harm to female reproductive organs and make a woman incapable of bearing any but feeble, sickly children. It was, contrary-wise, believed that the maturation of a female's body interferred with the performance of her mental functions. Gynecologists entered the arena with dire pronouncements. One, in 1905, went so far as to assert that academic application not only *decreased* the size of the female pelvis, but also *increased* the size of the heads of future offspring. A distressing situation indeed in those pre-anesthetic days (Bullough, 1973). Lest we feel such arguments are never heard in these enlightened times, I would offer from a 1971 book by Theo Lang: "Menstruation *must* have a retarding effect on a woman's physical and mental development."

The study of mathematics is vital to the intellectual development and the career progress of individuals regardless of sex. Non-required courses on this subject, however, are not now, nor have they ever been, heavily populated with females. Concerned teachers of mathematics are beginning to ask why this is so. They are not willing to accept biological reasons as total explanation of the difference.

Even if it can be shown that there is some biological advantage in the area of spatial ability for males, it will account for only a small portion of the phenotypic difference between males and females in the area of mathematics achievement. It cannot explain at all why a particular Susie performs less well in mathematical situations than a particular Sam does. The effects of environment explain the phenomena much more efficiently.

> Even at an early age, boys are expected to be interested in mathematics. Girls, on the other hand, though they may have equal ability, may be discouraged from learning by the prevailing idea that mathematics is a masculine field. (Lambert, 1960, p. 19)

Mathematics, in short, is incongruous to the female sex role.

We come to know, all of us, what *is* appropriate for males or for females by a transmission of expectations called socialization. Three different major theories are used to explain the process. One, building on the work of Skinner and of Bandura and Ross, states that differential patterns of reinforcement lead to the acquisition of sex-role-appropriate behavior. Social learning theorists advance the thesis that behaviors which are rewarded will be repeated and behaviors which are ignored or punished will tend to diminish in frequency. The behavior may be original with the subject or may be the result of imitating a model, the most influential models being those which are perceived as warm, powerful and nurturing. A second theory, derived from the work of Piaget, is the cognitive-developmental. Sex-role concepts are believed to be acquired in a manner similar to that by which any other concept is acquired. Kohlberg and Ullian (1974) believe that children do not accept the fact of gender constancy until about the age of six. Prior to that time, a child believes that his or her gender is determined by external appearances. As children come to accept gender permanence, they attempt to conform to the "proper" role. Whichever of these two theories is correct, it is clear that brighter children will tend to socialize earlier than their peers. According to the social learning theory, the socialization will occur earlier because fewer trials are needed for learning to take place; according to the cognitive-developmental theory, the child will pass through the stages more rapidly.

A third theory used to explain how children learn what behavior is appropriate for males and what for females is based on the work of Freud. By a process he called identification, the child incorporates as part of his or her own behavior the behavior of same sex models, especially the same-sex parent. The process, which takes place between the years of three and seven, is all but immutable. What Mommy does, or likes to do, becomes the benchmark by which her daughter decides what is appropriate behavior for herself, according to this theory of socialization. The Freudian theory has been expanded in the area of sex role acquisition by Parsons and Bales.

The message of what is appropriate behavior for males and for females, is, of course, no secret in our culture. We receive the message from the media, from advertisements, and from peers, parents and teachers.

Differential expectations according to sex and direct socialization by parents begin at birth (Rubin, Provenzano and Luria, 1976). The psychological mind set may be present even earlier. Wrinkled red-faced baby girls indistinguishable from similarly-featured boys are declared by parents to be finer-featured, delicate and cuddly in ways that conform to the cultural stereotypes on the basis of the color of the blanket wrapped around them. As he or she grows older, a child's environment is furnished according to his or her sex. In a study of the rooms of 96 children under 6, it was found that ruffles were present significantly more often in the rooms of girls. The absence or presence of ruffles will not have much impact, perhaps, on the mathematical development of females. Another finding of the study is more telling. There were only two dimensions on which a difference at the .001 level of significance was achieved, the presence of dolls and the presence of spatial and scientific toys (Rheingold and Cook, 1976). A survey of female mathematicians disclosed that scientific toys were the most difficult toys to get from their parents, even when the girls asked directly for them (Astin, 1975).

Parental expectations are often communicated directly as well as indirectly. Women in the Astin study reported that, in the "masculine" area of mathematics, their parents expected more from their sons than they did from their daughters.

> Parents who would suffer keen mortification if the boy for whose education they are responsible were in danger of being rejected by his college because of his failure in required mathematics will condone any shortcomings of the girl in that [area] with a deprecating "You know that one does not think so much of a failure in mathematics for a girl." (Martin, 1917, p. 396)

This observation by Emilie Martin, which appeared in the American Mathematical Monthly in 1917, is still valid 60 years later.

Parents can also play a major role in developing the later mathematical progress of their daughters. Helson (1971) found that almost every one of the women mathematicians she interviewed had grown up in homes with strong respect for learning and cultural values, and that as little girls, most of them had been rewarded for intellectual successes. Osen (1974) reports in a similar finding in a history of women in mathematics.

As important as parental attitudes and expectations may be, one cannot overlook the pressure of peers. Especially for the adolescent girl, peer pressure may be even more influential than that of parents (Lebold, 1976). Women who choose to continue in the study of this subject decried masculine by tradition receive less attention from their male peers (Luchins, 1976) and are often uncomfortable in social relationships with them (Fox, 1976). Mrs. Helen Andelin, mother of eight, author of Fascinating Womanhood and The Fascinating Girl, and director of Fascinating Womanhood courses, speaks directly to young women who might be tempted to fulfill their own intellectual interests without considering the possible effects on the men around them:

> Don't compete with [men] for scholastic honors in men's subjects. It may be all right for you to win over a man in English or Social Studies, but you are in trouble if you compete with him in math, chemistry, public speaking, etc. Men may admire women who excel in them but they are not apt to ask them for a date. Why? Because they have defeated them in their own field. (Andelin, 1970, p. 164)

One of the saddest facts about this quotation is that it often is true. In a study of 350 women mathematicians, Luchins (1976) found that not only had almost half of them been discouraged by teachers and by peers on the grounds that "boys wouldn't like them," but that these women received less positive attention from men as they advanced to higher levels. If it is to come, a young woman's interest in mathematics often crystallizes during the adolescent years. Eighty-eight percent of Luchins' respondents reported that their interest in mathematics had begun before the end of their high school years. Indeed by the time of high school graduation, 36 percent of the women had decided on a career in mathematics. Many more young women may have decided the choice just wasn't worth the price.

> In almost any age, it has taken a passionate determination as well as a certain insouciance, for a female to circumvent the crippling prohibitions against education for women, particularly in a field that is considered to be a male province. In mathematics the wonder is not that so few have attained proficiency in the fields, but that so many have overcome the obstacles to doing so. We can only speculate about the multitude who were dissuaded from the attempt. (Osen, 1974, p. 163)

There is little that mathematics educators can do which will directly influence parent-daughter or peer relationships. It is my sincere conviction, however, that those who train teachers and those who teach elementary and secondary students can have favorable impact on reducing a young woman's perception that mathematics is a masculine domain, but that this cannot be left to chance. Teachers, too, are currently products of their own socialization in this regard. They may believe that boys can usually be expected to be better math students (Ernest, 1974), or that even very bright girls at the elementary level will find mathematics a difficult subject or that success in mathematical endeavors is not appropriate and should not be rewarded for females (Levine, 1976).

What can be done to help teachers confront their unconscious beliefs with respect to the appropriateness of female achievement in mathematics? One successful strategy is to provide experiences which will cause these hidden assumptions to surface. Exercises which will do this are called values clarification exercises. These techniques can be used with little disruption in math methods courses, in student teacher seminars, and in in-service workshops for teachers. They are potent raisers of consciousness and force participants to examine the roots of their behavior in a way that a lecture on the career usefulness of mathematics or on the provisions of Title IX will never do. Values clarification exercises have this power because they often provide simultaneous cognitive and affective input.

Once the technique has been learned, variations of established exercises are easy. For example, one of the most easily adopted values clarification techniques is rank ordering. In this exercise, the participants are asked to listen to a situation and to place in order of preference the three choices presented. All alternatives must be ranked from the most acceptable to the least acceptable. None may be omitted. To get a feel for this technique, consider the following situation:

> Your district has just established an accelerated program for students gifted in mathematics. Enrollment will be very limited. In what order would you refer these three equally gifted students? The likelihood of their participation decreases as their position on the list is lowered.
>
> a. a deeply religious boy whose religion strongly encourages large families.
>
> b. a boy whose vision is very poor and who is expected to go blind before he is 20.
>
> c. a boy who is sullen and rude and does not study.

Having completed the ranking, consider this situation:

> Your district has just established an accelerated program for students gifted in mathematics. Enrollment will be very limited. In which order would you refer these three equally gifted students for the program? The likelihood of their participation decreases as their position on the list is lowered.
>
> a. a deeply religious girl whose religion encourages large families.
>
> b. a girl whose vision is very poor and who is expected to go blind before she is 20.
>
> c. a girl who is sullen and rude and does not study.

After you (or those with whom you do this exercise) have completed both rankings, some meditation on the similarities and differences in the rankings might be appropriate. If this is done as a group exercise, each individual should complete a ranking independently before general discussion takes place. The reasons given for the differences will provide information on the underlying assumptions which form the basis for decisions made in the real world. For a particularly difficult task, try rank ordering all six of the candidates for the special class.

A second, related category of exercises are Forced Choice situations. To present a forced choice exercise, one describes a situation with two approximately equally attractive (or repulsive) alternatives and asks participants to choose one alternative. Abstention is not allowed. The choices made need not be publically shared, but the individual must come to a decision as to which is his or her choice. An example of a forced choice situation:

> You can refer children from your elementary classroom in each of the five categories named. Only one child can be referred for each of the five. Would you refer a boy or a girl whom you believe equally in need of special help?
>
> a. the child is withdrawn
> b. the child is gifted but has a mild behavior problem
> c. the child has reading difficulties
> d. the child cannot grasp arithmetic concepts at all
> e. the child is aggressive.

This situation is based on research conducted with 140 elementary (K-6) teachers. Gregory (1977) found that in every case, boys were more likely to be referred than girls. The difference for reading referrals did not reach significance, but in every other case, the likelihood of referral of boys was significantly greater. Indeed, the difference in likelihood of referral for the child with severe arithmetic disability was at the .023 level. A girl with such deficiency had less than a 50/50 chance of being referred for help. A boy had about a 40 percent chance of being referred. The findings are even more depressing than they seem at first glance. The teachers in the study were not given a forced choice. They received a set of five anecdotes describing the children and were asked simply to state whether they would or would not refer each child. There was no limit on the number of children they could refer. Teachers were willing to ignore severe arithmetic problems in both boys and girls but were even more inclined to consider such problems unimportant for females.

Another values clarification exercise which is easy to incorporate and fun to do is imaging the ideal—blue-skying, as it is sometimes called. Ask a group to list the qualities of an ideal female student. Then, on another piece of paper, list the qualities of an ideal male student. Then have them list the qualities of an ideal math student. Compare the lists.

This exercise, too, is based on a research study. In the study reported by Sadker and Sadker (1974), the ideal female student was perceived to be:

appreciative
calm
conscientious
considerate
cooperative
dependable
efficient
mannerly
mature
obliging
poised
sensitive
thorough

The ideal male student was perceived to be:

active
adventurous
aggressive
assertive
curious
energetic
enterprising
frank
independent
inventive

One of these lists is apt to correspond much more closely than the other with the list developed for "ideal math student." Perhaps in the light of the lists developed in your own reenactment, it might be well to consider the factors that Rossi (1972) has identified as characteristic of scientists of note:

1. High intellectual ability
2. Intense channelling of energy in one direction
3. Extreme independence
4. Apartness from others without guilt or longing

Females in our culture have traditionally been socialized to conform rather than to contradict, to be popular rather than to excel academically, and to accept as the most appropriate goal, raising a family rather than pursuing a career. The results of successful socialization according to this pattern is not apt to encourage achievement in a masculine subject during the high school years. As Rossi states:

> We must encourage (in young women) the cultivation of the analytical and mathematical abilities science requires. To achieve this means encouraging independence and self-reliance instead of pleasing feminine submission in the young girl, stimulating and rewarding her efforts to satisfy her curiosity about the world to the same extent her brothers' efforts are, cultivating a probing intelligence that asks why and rejects the easy answers instead of urging her to please others and conform unthinkingly to social rules. A childhood model of the quiet, sweet, "good" girl will not produce many women scientists or scholars, doctors, or engineers (1972, p. 79).

It will not produce creative mathematical thinkers either.

Is it possible to bring about change in the pervasive stereotypes of mathematics as a masculine domain? Yes, I believe it is. I am not so naive as to believe it will be an easy task, however. A stereotype is an assumption that because individuals are alike in some ways they are similar in many others. There are usually some readily available examples of the "truth" of a stereotype which can be trotted forth whenever one is under examination. What those who prefer such examples ignore, of course, is that no amount of particular examples can prove the general case. Stereotypes are a type of shorthand which we use to encapsulate the messages of the world into more easily-handled bits of information. They are dangerous for the same reason. Stereotypes ignore individual strengths and weaknesses, rights and duties and make judgments based on class membership. In our culture, the stereotype that women aren't good at math—or that a woman who does not conform to this stereotype is somehow not really feminine—is well-entrenched. It will not be eradicated easily.

As one sets about to make change in an attitudinal area, it is well to remember that dramatic change is most unlikely, that rapid change will probably not occur and that the increments of change may well be small and slow in coming. One model which may help explain some of the discomfort as one sets about attitudinal change postulates seven steps in the process of attitudinal change (McCune, et al., 1977).

The first step in the process is Shock and Surprise. When people become aware of the inequity of a condition they have long accepted as a given, they usually experience this stage quite vocally. It is followed, comparatively rapidly, by a second stage, Disbelief. Commonly heard statements from counselors, administrators and teachers in this second stage are variations of, "Well, that isn't true at this school." Commonly heard statements from professional educators in this stage are variations of, "Well, if the researchers had chosen a larger sample, or had run the XYZ test, or had not been biased to start with, the results would be different."

The third stage is the most painful for many of us. It is the stage of Guilt. All of us probably feel some guilt with respect to issues of sex bias. After all, we are all products of our culture, too. The danger of this stage is that one can become so immersed in guilt that one remains mired here. If, because of past actions, one allows himself or herself to feel unworthy to work for change, a vote is, in effect, cast for the status quo. It is legitimate to feel guilt; it is also legitimate to forgive oneself and move on to more positive actions for change. One who attempts to bring about change in others should be aware that this will be a most painful time and will need to be prepared for the next level in the change process, the stage of Projection of Guilt.

When an individual has borne all the guilt he or she can comfortably bear, when beating the breast is no longer fun, the usual course of action is to look around for someone else to blame. A person in this stage makes statements such as, "My mother always told me --;" "The counselor I had --;" "No girl who is trained in math to that level is going to find a husband." Projection can become such a comfortable stage that one is tempted to sit back smugly and toss the blame around with great abandon. Tossing blame around has never been an effective technique for bringing about social change, however.

You may have noted that these first four steps have concentrated on the Feeling level. In the fifth step, one beings to use rational means to solve the problem by Identifying Specific Problems. It is at this step, and not before, that reporting research becomes a useful strategy. While research is sometimes of value in arousing surprise and thus initiating change, it is not until feelings have been processed that an individual is ready to think about effective resolutions for a problem situation. A useful technique for identifying specific problems is called Force Field Analysis. The analysis is undertaken by listing on one side of a line all the forces in a given situation which are working towards change in the desired direction and listing on the other side all the forces that are working against that change. The more specific the forces listed, the more valuable the technique is likely to be. In addition to helping identify the specific problems which prevent girls from excelling in mathematical endeavors, for example, the technique moves the person considering the problem to step six of the stages of change, Formulating Action Steps.

The sixth step is also in the thinking dimension. It is usually wise to try to weaken the "against" forces rather than to strengthen the "for" forces. A strengthening of the "for" forces can cause increased resistance. The best alternative, when it is available, is to change a force against the change to a force for change.

The final step, the only step which deals with Doing, and the only step with perceivable results, is the integration of change into ongoing behavior. This step takes a long time to bring about, and the slowness of the process can be extremely frustrating. Change can happen however, and I would like to describe briefly some specific action steps that have been found helpful in increasing female participation and pride in mathematical endeavors.

Be Aware of Your Expectations

Never accept less than a given child's best work. It is demeaning, degrading, and disastrous to say or imply, "Well, you're a girl, I don't expect you to do as well in math as boys do," or "You're a boy, most boys read less well than girls do." Expect the best possible performance from every student, regardless of sex.

Help Students Learn to Learn Mathematics

The state of the art of mathematics today precludes the naive assumption that we presently know all there is to know about mathematics theory or applications. A student who is taught to respond to mathematical situations with an automatic algorithm is hampered in future learning in the subject. Placing at least as much emphasis in the teaching of the why of the process as well as the how will provide students an opportunity to develop problem-solving skills which go beyond the immediate situation and will stand them in good stead when they encounter situations in the future.

Provide Biographies of Female Mathematicians

Traditionally, biographies in science and mathematics have been of the "male only" variety. There are at least two readily available books on the contributions of women to math and science (Osen, 1974; Perl, 1978). Do have those available, as well as your old favorites.

Examine the Career Relevance of Mathematics for Young Women

Teachers of mathematics have long known that at least three years of high school mathematics are required for entrance into engineering, science and mathematics majors at most colleges. They also realize, better than most, that even for entrance into the social and behavioral sciences, long the haven of the math-wary, mathematics is now required.

Male high school students believe that mathematics will have career relevance. Female high school students are not as convinced that this is the case (Hilton and Berglünd, 1974; Ernest, 1976; Fennema and Sherman, 1977). Those not interested in mathematics or science as a career may not recognize a need for mathematics for careers in teaching or the social sciences.

Mathematics teachers and/or counselors might collect literature on mathematics dependent careers for display in their classrooms and make it available to other teachers and to student groups. A list of speakers on careers for which mathematics is essential might be compiled as a class project. Two free booklets, "Careers for Women in Mathematics," and "Math in High School You'll Need for College" (prepared by the MAA with the cooperation of NCTM) may be obtained from the Mathematical Association of America by writing them at 1225 Connecticut Avenue, NW, Washington, DC. Permission has been given for local reproduction of "Math in High School." It might be worthwhile considering putting a copy of this leaflet in the hands of every Algebra I student in your school.

Identify Resources for Personal Growth

There are a growing number of research studies which relate directly or indirectly to the issue of women and mathematics. Several of them are listed in the bibliography of this paper, and many others are to be found in other papers in this volume. Do a little spare time reading; get the information you need to bolster your own arguments and to formulate your own action steps.

Encourage the Development of Spatial Skills

Adults use spatial skills in a wide variety of settings, from parking a car to planning a new kitchen. These skills are important in occupations as divergent as dressmaking, architecture, surgery and draftsmanship and in everyday life and scholastic endeavors as well.

A lack of confidence with respect to these abilities has not only an immediate effect but also, because it can affect career choice, a long-range impact as well. Even if it is proven that there is a biological component to the development of spatial visualization, there is much that can be done to foster further development of these skills. Aiming objects at a target, throwing and catching balls, building with a variety of elements from giant hollow blocks to Lincoln logs, legos and erector sets, engaging in sand and water play and climbing on a jungle gym or running across a field, may all help develop spatial skills. No child should be discouraged from engaging in any of these activities because it is not "ladylike."

Including in the mathematics curriculum units with spatial skill-building possibilities such as tangrams and map and compass reading is a practical and mathematically sound way to encourage the development of spatial skills.

Use Testing Results Carefully

Learn to take all results of standardized tests used in career planning with a large grain of salt. Many of these tests contain biased items and many of them were standardized on populations much unlike the students who take them today. A score on some item such as "mechanical interest" which would be considered low for a male student is a significantly high score for a female student. Some counselors believe it is wise to discuss with the student the norms on both scales for tests which are separately normed by sex. It is rather universally accepted that the numerical outcomes of a test should not be considered as the final word in the decision-making process. Allow a considerable margin for testing error and trust your own good sense as well as the test results.

Invite Guest Speakers to Mathematics Classes

Role models are powerful influences on human behavior. Even in audiovisual presentations, the presence of a female role model has a positive effect on high school women (Plost and Rosen, 1974). The effect of in vivo models is even stronger (Casserly, 1975; Levine, 1976). In a 1976 National Science Foundation project designed to assess strategies used to increase women's participation in mathematics and science, the presence of role models during the high school years appeared to be the most effective (Fox, 1976). Early exposure to adults who lead satisfying lives both as women and as professionals in mathematics-dependent careers will provide proof of the appropriateness of mathematics as a field of study for females as well as for males (Westervelt, 1975).

Belief in the stereotype of mathematics as a masculine endeavor becomes a self-fulfilling prophecy. Currently, great numbers of women do not enter the field of mathematics, so mathematicians tend to be men, engineers tend to be men, and the fiction that mathematics is an esoteric science women cannot understand is reinforced. Belief in the stereotype "encourages the notion that to enjoy mathematics in its many forms is to be, in some obscure way, at variance with one's womanhood" (Osen, 1974, p. 165). Show young women this need not be so.

Encourage Female Students to Encourage Other Female Students

It is not surprising that peer role models are very effective. For a young woman bright in mathematics to meet other young women who share a similar talent may well provide the support she needs to continue her pursuit of the subject. The use of mathematically competent and confident juniors and seniors to encourage elementary school students and younger secondary school students is particularly powerful. If the academically successful peer models are popular with other students, both male and female, their testimony will be particularly useful.

Help Young Girls Accept Their Mathematical Talent

In one study, female mathematicians were asked why so few women choose mathematics as a career. Seventy-three percent of the respondents stated that the major reason for the relatively small number of female mathematicians was that mathematics is not considered "feminine" (Luchins, 1976).

When there is potential for conflict between academic achievement, particularly in "masculine" areas, and popularity, achievement tends to suffer (Fox, 1976). Whether the concern is valid or is an incorrect perception on the part of the young woman, the effect is the same. Indeed, young women tend to underrate their ability in mathematics, even when it is exceptional (Maccoby and Jacklin, 1974; Robitaille, 1976; Ernest, 1976).

A teacher who can help a young woman accept her mathematical talent as a legitimate aspect of her personhood does more than can be put on paper to foster the young woman's actualization of potential growth.

Examine Descriptions of Extra-Curricular Activities

A young woman who is uncertain about joining the math club may be even further put off if the descriptions read "A member of the club ... he ..." Try to make extra-curricular activities appealing and appropriate for both sexes. Make your promotional literature a model of sex fairness.

Advise Textbook Committees of Publisher's Guidelines

Every major publisher has, within the last three years, published and distributed guidelines for their authors on promoting sex-fair treatment of males and females in textbooks. Be sure your local, county, or state textbook selection committee is apprised of your concern in this area and has a copy of the pertinent pamphlets. Most publishers will send you a copy of their guidelines at your request.

Work with Counselors

In an ETS study of female high school students gifted in mathematics, Casserly (1975) found that even these young women were often counseled out of advanced mathematics courses. Counselors and/or teachers suggested that serious study might detract from the fun of senior year, cautioned that the student might spoil her scholastic record, or expressed fears that a woman trained in mathematics or science might steal a job from a man.

In the past, assumptions about which behaviors and goals are appropriate for young women has restrained both teachers and counselors from suggesting additional courses in math, and has prompted them to question the suitability of such courses when they have been selected. Counselors who are convinced of the worth of the study of mathematics, even from a pragmatic career perspective, will prove a valuable ally to mathematics teachers seeking to encourage students to take advanced, non-required courses.

Once a student has expressed an interest in mathematics of a mathematical related career, someone must act as a mentor, helping the student clarify her goals, choose a college, fill out forms, and attend to many other details. Finally, many teachers have gone beyond the call of duty and have contacted parents who were unaware of the potential of their daughter, or who regarded mathematics as an inappropriate career choice. Counselors may be pleased to put their training to use in these ways.

Be Aware of Available Resources

Personnel of the State Department of Education in many states are very interested in the issue of sex-fair education. People who have such an interest might be found in the division of mathematics, the division of equal educational opportunity, or the division of counseling and guidance. Contact your State Department of Education and locate those interested in this issue and form a network of support for yourself. Try your county or district offices also. In either case, a good person to ask for leads might be the Title IX coordinator.

In addition to funding for research and development, there are provisions on the federal level for two types of technical assistance operations to help allay sex bias. The first type, General Assistance Centers, work with school districts on request across a wide geographic area. Funded under Title IV of the Civil Rights Act of 1964, GAC's have worked for several years with school districts to implement desegregation plans. Three years ago, the scope of their activities was increased to include the dissemination of information on Title IX and giving assistance to school districts in providing a sex-fair education. The location of the General Assistance Centers may be obtained from any regional Office of Education (see Table 1). A second type of government-funded group which may be able to help your district encourage the increased participation of girls in mathematics are Sex Discrimination Institutes. Regional Offices of Education can also provide information on the location of these institutes. Finally, regional Offices of Education distribute information on the existence of any Women's Educational Equity Act funded projects which are concerned with women and mathematics, and on the ways materials developed in those projects can be obtained.

Table 1

DEPARTMENT OF HEALTH, EDUCATION AND WELFARE REGIONAL OFFICES

Region I

(Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island, Vermont)

RKO General Building
Bullfinch Place
Boston, MA 02114
(617) 223-6397

Region II

(New Jersey, New York, Puerto Rico, Virgin Islands)

26 Federal Plaza
New York, NY 10007
(212) 264-4633

Region III

(Delaware, D.C., Maryland, Virginia, West Virginia, Pennsylvania)

Gateway Building
36 and Market Streets
P.O. Box 13716
Philadelphia, PA 19104
(215) 596-6772

Region IV

(Alabama, Florida, Georgia, Kentucky, Mississippi, North Carolina, South Carolina, Tennessee)

50 Seventh Street, NE
Atlanta, GA 30323
(404) 526-3312

Region V

(Illinois, Indiana, Minnesota, Michigan, Ohio, Wisconsin)

300 South Wacker Drive
Chicago, IL 60606
(312) 353-7742

Region VI

(Arkansas, Louisiana, New Mexico, Texas, Oklahoma)

1200 Main Tower Building
Dallas, TX 75202
(214) 655-3951

Region VII

(Iowa, Kansas, Missouri, Nebraska)

Twelve Grand Building
12th and Grand Avenue
Kansas City, MO 64106
(816) 374-2474

Region VIII

(Colorado, Montana, North Dakota, South Dakota, Utah, Wyoming)

Federal Building
1961 Stout Street
Denver, CO 80202
(303) 837-4345

Region IX

(Arizona, California, Hawaii, Nevada)

Phelan Building
760 Market Street
San Francisco, CA 94102
(415) 556-8586

Region X

(Alaska, Idaho, Oregon, Washington)

Arcade Plaza Building MS 508
1321 Second Avenue
Seattle, WA 98101
(206) 442-0473

Raise the Awareness of Others

Let people who have clout know that this is an important issue with personal and economic ramifications for the young people you teach. Much discouragement comes not from overt anti-female attitudes as much as from a lack of awareness of the facts of life with regard to the work future of women. It is important that all who work with young women realize that 90 percent of the young women now in school will work an average of 45 years; those who marry, only 25 years. Those years should be as satisfying as possible for all young people, both professionally and personally.

Become Aware of the Effects of Math Anxiety

Many men and women suffer such a fear of mathematics that they are all but paralyzed when confronted with even the simplest of situations involving the application of mathematical concepts. A cause as well as a symptom of math anxiety is the unwillingness to take courses in mathematics. Teachers who explain to math anxious students that fear of the subject is far from uncommon, that it is a phenomena that has affected students from kindergarten to the doctoral level, and that it can be conquered, are both honest and helpful. Focusing on the roots of the anxiety, individually or through a discussion group, is often effective. Such a discussion group might be offered as an after-school workshop or as a mini-course.

Towards Tomorrow

Some of these action steps may be of help in your situation. Attitude on behavior change is never easy, but it does occur. But they do happen. Mathematics is not only a masculine domain. Many people in the past and in the present believe it need not be. For example:

> The gifts of nature are alike diffused in both (men and women) ... All the pursuits of men are the pursuits of women.
>
> Plato, The Republic, 4th
> Century BC

> I say it to thee again, and doubt never the contrary, that if it were the custom to put the little maidens to the school, and they were made to learn the sciences as they do to men-children, that they should learn as perfectly, and they should be as well entered into the subtleties of all the arts and sciences as men be.
>
> Christine de Pisan, La Cite
> des Dames, 1407

> Perhaps she should learn arithmetic before anything else; for nothing is more obviously of general use, nothing needs longer practice, and nothing gives more opportunities for mistakes than accounts.
>
> Rousseau, Emile, 1762

Intellect is not sexed.

Sarah Grimke, The Equality of
Sexes and the Condition of Women,
1838

Like Charles Lamb, who atoned for coming late to the office in the morning by going away early in the afternoon, we have, first, half educated women, and then, to restore the balance, only half paid them.

Thomas Higginson, Women and the
Alphabet, 1900

When a person of the sex, which according to our custom and prejudices, must encounter infinitely more difficulties than men to familiarize herself with these thorny researches, succeeds nevertheless in surmounting these obstacles and penetrating the most obscure parts of them, then without doubt she must have the noblest courage, quite extraordinary talent and superior genius.

F. Gauss, Speaking of Sophie
Germain, Quoted in E.T. Bell
Men of Mathematics, 1937

No sane person now considers it any more "unwomanly" or more "monstrous" for them to study or teach mathematics than for them to teach music or needlework.

H.J. Mozans, Women in Science,
1913

The unfounded assumption that mathematics is more a subject for males than a subject for females is to be vigorously opposed.

NACOME Report, 1975

The inability of woman to succeed at mathematical endeavors is a bit of hallowed mythology in our folklore, but there are those who seek to change that. Join us. Encourage each mathematically talented Susie you know to excel in mathematical endeavors and to be proud of her ability.

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SEX, SEXISM AND ANXIETY IN MATHEMATICS

by John Ernest

The mathematics department at my home institution, the University of California at Santa Barbara, consists of 47 full-time faculty. Not one is a woman. These 47 men are served by a support staff of seven secretaries and typists, who are, you guessed it, all women. Four years ago I, together with ten enthusiastic freshmen, made a study of sex differences in mathematics education and employment. At that time the UCSB mathematics department consisted of 32 full-time faculty, all male. Thus while over the last four years my department has increased in size by 15 members, it continues to have the character of a men's club. I wrote an essay on the results of my freshmen seminar of four years ago, and gave it the provocative title "Mathematics and Sex."* I now know why sex is used to sell everything from records to automobiles. The essay has been widely read, I'm sure largely because of the deceptive title.

Perhaps the most frequent comment I received about this essay, always given "tongue in cheek," was that the reader was disappointed that there was really so little sex in the essay. Perhaps because of these many gibes at my misleading title, I have come to realize that sex is indeed a very real issue in any presumably comprehensive discussion of the problem of the disproportionate representation of women in the mathematics field. Therefore, with considerable trepidation, I would like to venture a few remarks on this sensitive issue. I do so not because I have a lot of data to share with you, but because I want the issue to be explicitly raised, in the hopes that someone else, preferably a respected and trusted female mathematician, might begin to collect comments about sexual and interpersonal difficulties experienced in the pursuit of mathematics.

One woman mathematician told me of a meeting she attended on the dearth of women in mathematics and science. During the formal sessions, the usual issues involving role models, lack of encouragement and job discrimination were discussed extensively. But not until the informal discussions during the breaks did people begin sharing their personal "horror stories" involving various male teachers or colleagues. I know of one woman mathematician who had to change her Ph.D. thesis advisor. His never-ending attempts at seduction made it impossible to maintain a productive academic relationship.

Underlying the sexism pervading the mathematics profession are some very fundamental problems involving how men and women relate to each other. We cannot work well together as colleagues until we break away from the stereotypical roles which keep us from relating fully at many levels. We men have grown so used to these artificial roles that we find nothing unnatural about having an all-women staff type the letters for an all-male faculty. We take for granted that these women will also take time to make us coffee. We never stop to question the fact that we call the staff members by their first names, while they call us Dr. this and Professor that, always using our last names. It is still common practice to refer to the members of the staff as "the girls." Sure we can make changes in textbooks and employment procedures, but the basic sexist patterns will continue until we change the underlying attitudes of male mathematicians in their relations with women in general.

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*American Mathematics Monthly, Vol. 83 (1976), 595-614.

I do not wish to deny the issue of sexuality in relations between men and women. And I'm certainly not suggesting that sex be prohibited in professional mathematical circles. That would be a clearly unattainable objective, even if it were considered desirable by some (not by me). Quite to the contrary I wish that issue could be examined openly and honestly so that we can better understand how to ensure that the sexual aspects of our interaction will not interfere with our ability to work together as professional colleagues on a totally equal basis.

I'm often asked to summarize the essay, "Mathematics and Sex" in one or two sentences. I'm asked, "What is the basic problem causing extreme sex differences?" I usually reply that it is a multifaceted problem involving many interrelated factors, such as societal attitudes, parental pressures, peer group expectations, lack of role models, sexist textbooks, bad teachers, poor career counseling, etc., no one of which can be isolated as primary. However, if pressed for a single underlying "cause," I would have to answer somewhat philosophically that I am the problem. I am the product of a sexist culture and I carry with me many engrained and unexamined attitudes which color all my relationships, and which I take with me every time I enter the classroom. By extension, all of us who are teachers, with our awesome capability to influence the young, must realize that first and foremost the problem is within ourselves. We have the responsibility to first examine our own sexist attitudes, to first change our own patterns, before we can hope to enter the classroom and not do irreparable harm. And here I am talking about both male and female teachers, for we are all burdened with this sexist baggage. In our questionnaires we asked students to explain what forces brought about their strongly negative or their strongly positive attitudes towards mathematics. The most frequent responses referred to either poor, sarcastic teachers or to inspiring, encouraging teachers.

If as males we feel intellectually superior to the women in our lives, or if we have not learned to relate to women as total individuals but only as objectives of sexual seduction, then we cannot help but do great damage in the classroom, no matter what our good intentions. If as females we feel apologetic for our intellectual skills, or if we feel anxious and unsure of ourselves in handling quantitative problems, or if we feel there is something basically unfeminine about science and engineering, then these feelings will certainly be perpetuated by the sensitive students in our classes.

Thus our first responsibility is to work on ourselves, carefully uncovering our own sexist prejudices, through some serious introspection. Just as the psychiatrist must first go into analysis before he or she can presume to counsel others, so must we first eliminate our own sexism before we can effectively ferret out the sexism prevalent in our educational institutions. After we become aware and sensitized to the many sexist assumptions of our outlook, and here I mean something much deeper than learning to mouth a few feminist slogans, then we can begin to deal with sexism in the classroom honestly and with understanding. For example, we all know how it is possible to turn a mathematical error in a text to pedagogical advantage as we make the students aware of the mistake. Similarly when traditional male and female roles creep into various problems or illustrations of the text, the alert teacher can seize that opportunity to make students aware of how such stereotypes are subtly perpetuated. Thus the concerned teacher who works conscientiously to create a non-sexist classroom environment can even utilize sexist tainted educational materials to generate an awareness of the stereotypical thinking common to the area of mathematics. On the other hand, no amount of laundering of mathematical texts will afford any protection against the enormous damage done by those teachers who are either unaware of, or insensitive to their own sexist attitudes.

Speaking of the damage which instructors can do to students, brings me to the topic of "mathematics anxiety." I sometimes get the feeling that some students develop more anxiety about the mathematics instructor, than they do about the subject matter itself. A male mathematician, scoffing at the whole notion of mathematics anxiety, jokingly told me that his main contribution to the area is that he is one of those who causes the anxiety. Indeed some mathematicians can be very creative in finding ways of generating anxiety. We all know of cases where a certain instructor is thought by many to be a great genius in mathematics simply because no one can understand his lectures. His ego gratification is unfortunately bought at the expense of making everyone else feel like an idiot. I certainly get anxious when attending a research lecture which is apparently in the English language, but where I don't understand anything. Then there are those instructors who are so unsure of their own ability to understand and explicate the material that the students themselves lose all self confidence. There are of course almost as many methods of generating anxiety as there are teachers. There is the one who delights in pouncing on students with unexpected questions, "to keep them awake." A good diet of unannounced quizzes can also be quite effective. Still nothing is quite as effective as the sarcastic and humiliating remark, when someone is brash enough to ask a "dumb" question.

Of course research mathematicians themselves experience anxiety when working on deep problems. I also know that some mathematicians experience some anxiety whenever the topic of "math anxiety" comes up. Some of these anxieties may arise merely from faulty communication. Among those interested in the subject we find people with quite varied backgrounds, from the social scientist concerned with sociological and psychological issues, to the mathematics educator concerned with teaching basic mathematical skills, to the research mathematician concerned with creativity at the Ph.D. level. With such diverse perspectives, some misunderstandings are inevitable. In the interest of minimizing these differences, and fostering more dialogue between the mathematician and the non-mathematician I'd like to play the role of a gadfly for a moment, and indicate some of the questions, doubts, misgivings, suspicions and anxieties that some mathematicians have expressed to me about the recent surge of interest in "math anxiety."

Fact: Large numbers of students are graduating from high school as mathematical illiterates. In particular, a scandalously small proportion of women entering college are prepared to take the basic beginning science courses in calculus, physics, chemistry or engineering. Just when we need to get back to basics, will discussions about our feelings towards fractions be one more frill which diverts students from courses with real content? Will the time and energy devoted to the issue further dilute the basic core curriculum? Will these programs quickly and effectively channel the mathematical drop-outs back into a serious and demanding course sequence?

In regard to the well-known dearth of women in disciplines depending on mathematics, will "mathematics anxiety" serve as still one more cop-out that women may use to justify why they do not persevere and excel in mathematics? Thus while these programs may make many women aware of their avoidance behavior, quite possibly because of undue anxiety, there is also the danger that there will be others who will find still one more rationalization to justify opting for less demanding courses. I'm sure there are women (and also men) who do need encouragement and reassurance that they can handle fractions. I'm equally sure there are other women who will find this a particularly irksome form of sexism—a paternalism which fosters very limited aspirations.

Of course many men exhibit math avoidance. There are plenty of male loafers at college, who enjoy a university life consisting mainly of surfing and beer parties. I'm sure many fathers have given their sons some very tough talks: "I'm tired of you trying every soft course and easy grade you can find. I want to know what you plan to do with yourself when I stop supporting you. What are you going to do for a living? How are you going to support a family? I don't want to see any more basket weaving courses. Figure out what you want to do in life and then get your ass in gear." Young men get this kind of tough advice because in our culture every man is expected to be the primary financial support of the family unit. But how many fathers or mothers, or teachers, or counselors would take such an attitude towards women students? An eclectic program of diverse courses with no career orientation is accepted because there is that unwritten assumption that the basic economic support will still come from a husband. How many parents are ready to get equally tough with their daughters: "Don't get yourself in that traditional bind where the quality of your life depends entirely on a possible future husband. Now is the time to determine exactly what you want to accomplish with your life, independently of any present or future relations with men. Then persevere in mastering the basics of your field, and move onto higher achievements. You can do it. Let's not hear any more of those tired, lame excuses—like it's not feminine to succeed—or that you are too timid to ask questions—or that women have no need to know algebra—or even that you have some kind of psychological block—you do have the ability. Don't let it go to waste — Take hold of your life — Get on with it."

I am not saying this kind of "get tough" advice is appropriate for all women—just as it is not appropriate for all men. But many young men who have perhaps dallied too long with surf boards and rock music, do get this kind of strong talk—from their fathers—from their teachers —and sometimes even from their friends. Our concern with anxiety, fear and insecurity should not lead us to exhibit any lower expectations for our daughters than we have for our sons. As teachers, let us make no fewer demands of our female students than we do of our male students.

Often anxiety is to be anticipated in intellectually demanding situations. I was anxious when I was writing this paper, and I am anxious now as I present it. Both women and men must come to accept such occasional anxieties and not let them become debilitating. The aim should not be to avoid all anxiety-producing situations. Indeed it is just such avoidance behavior which has caused many people to skirt all mathematics courses. Rather let us learn to understand those anxieties, to cope with them, and keep them in proper perspective.

There are many mathematicians who are observing these various innovative mathematics anxiety programs, and who are trying to evaluate them with an open mind. For most of them the primary criterion is whether these efforts result in a significant increase in participation, particularly by women, at all stages of mathematical and scientific education and employment.

I, for one, am convinced that these programs can be beneficial, not as a panacea, but as one component of a total program aimed at increasing female representation in the scientific and technical fields.

SEXISM IN MATHEMATICS TEXTBOOKS

(Some Observations and Suggestions)

William H. Nibbelink and Matadial Mangru

There are several strategies which might be used for eliminating sex-bias from mathematics textbooks. Among them are (1) largely removing references to persons, (2) using gender-free labels when referring to persons and (3) using equal numbers of males and females randomly assigned to roles. When it comes to ridding curricular materials of sex stereotyping, mathematics enjoys a luxurious position in that almost all story problems involving people are about ficticious characters. Thus, if something should be done, it could be accomplished by a single mandate from a managing editor.

The purposes of this paper are (1) to examine trends in textbooks over the past half century, (2) to discuss the nature of the sexism that persists in textbooks and (3) to suggest the strategy for eliminating sex-bias from textbooks which holds the most promise.

To find trends relative to story problem practices, five periods are examined: (M-30s) the middle-1930's, (M-50s) the middle-1950's, (M-60s) the middle-1960's, (E-70s) the early-1970's and (L-70s) the late-1970's. Data for each period are based on the three series which enjoyed widest classroom usage by children in Iowa during that period. Thus, copyright dates for a series would usually be a few years earlier than the corresponding period under study. For example, L-70s would be concerned with books having middle-1970's copyright notices. Attention is also restricted to grades 3-6.

Each entry in Table 1 gives the number of problems (by period and classification) that a child could expect to find in his/her books covering grades three through six. For example, the entry 223 for "male in male role" in the "M-60s" comes from 62 such third grade problems, 60 fourth grade problems, 51 fifth grade problems and 50 sixth grade problems. The bottom row, labeled "sex-role mismatch" presents for each period the number of problems a child could expect to find in which either a male is placed in a female role or a female is placed in a male role. Which role (male, female, or neutral) is assigned to a problem setting is determined by which persons typically hold the position named or engage in the activity described. For examples, baking a cherry pie is classified as female role, driving a gravel truck as male role, and buying a pack of gum as neutral role. In some cases, it may be interesting to the reader to think of three periods. The first two columns (M-30s and M-50s) may be viewed as samples of the "Old Math" period, the next two columns (M-60s and E-70s) as samples of the "New Math" period and the last column (L-70s) as a sample of the "Early Aftermath" period.

Table 1

Distribution of Story Problems for Five Periods for Grades Three through Six Based on the Three Series with the Widest Usage in Iowa

	Period				
Grades 3-6	M-30s	M-50s	M-60s	E-70s	L-70s
Total Number	2440	2225	1632	1641	1357
Not About People	869	995	625	748	836
About People	1571	1230	1007	893	521
About Children	1399	1087	785	663	328
About Adults	172	143	222	230	193
About Males Only	915	677	563	480	273
About Females Only	504	421	340	381	215
About Both or Either	152	132	104	32	33
Male in Male Role	332	271	223	253	145
Male in Female Role	32	10	3	0	9
Male in Neutral Role	551	396	337	227	119
Female in Female Role	249	186	133	138	57
Female in Male Role	4	4	5	2	21
Female in Neutral Role	251	231	202	241	137
Sex-Role Mismatch	36	14	8	2	30

Several changes over the time span sampled by Table 1 are conspicuous:

1. The number of story problems has been cut by almost half, as shown by row 1.

2. The number of problems not about people has remained fairly constant, while the number of problems about people has been cut by about two-thirds, as shown by rows 2 and 3.

3. The number of problems about adults has remained fairly constant, while the number of problems about children has been cut by about three-fourths, as shown by rows 4 and 5.

4. There is a trend toward even numbers of males and females in problems, as shown by rows 6 and 7. (Row 8 accounts for problems in which the genders of persons are not specified or in which both males and females are portrayed. In all cases, such problems involved neutral roles.)

5. There has been and still is a relatively low frequency of males in female roles, as shown by rows 9, 10, and 11. For L-70s, the child who looks at all problems in the math books over grades 3 through 6 may expect to find a male in a female role about once every four months.

6. There is a recent increase in the number of problems which place a female in a male role, as shown by row 13. However, the number could hardly be called high. The L-70s child who looks at all problems in the math books over grades 3 through 6 may expect to find a female in a male role just over once every two months.

7. In summary, the reluctance to cross traditional role boundaries, while less than before, is still evident, as shown by row 15.

The data presented by Table 1 show that to a substantial degree both the number of and the proportion of problems which might be of interest to a committee concerned with sex stereotyping have been reduced. This results from fewer problems in general, people being replaced by little birds and other animals whose feathers or fur defy assignments of gender, and by a recent wider usage of presenting story problem information by pictures and diagrams.

If it is assumed that the number of problems in books today is adequate, and that in prior times children were merely assigned a representative sample of problems in books, then Table 2 gives a clearer picture of practices and trends over the periods under consideration. Table 2 presents the percentages of problems falling into the various categories for each period.

Obvious from Table 2 are the following statements regarding the proportions of problems in given categories to all problems:

1. Problems about people have decreased and problems not about people have increased substantially.

2. Problems about children have decreased and problems about adults have increased.

3. Both problems about males only and problems about females only have decreased, but the ratio of females to males is approaching one.

4. Problems presenting a sex-role mismatch have increased, but remain far short of showing a random assignment of sex to role.

Table 2

Distribution of Story Problems for Five Periods for Grades Three through Six (Entries are percentages based on all problems)

Grades 3-6	M-30s	M-50s	M-60s	E-70s	L-70s
Not About People	35.6	44.7	38.3	45.6	61.6
About People	64.4	55.3	61.7	54.4	38.4
About Children	57.3	48.9	48.1	40.4	24.2
About Adults	7.0	6.4	13.6	14.0	14.2
About Males Only	37.5	30.4	34.5	29.3	20.1
About Females Only	20.7	18.9	20.8	23.2	15.8
About Both or Either	6.2	5.9	6.4	2.0	2.4
Male in Male Role	13.6	12.2	13.7	15.4	10.7
Male in Female Role	1.3	0.4	0.2	0.0	0.7
Male in Neutral Role	22.6	17.8	20.6	13.8	8.8
Female in Female Role	10.2	8.4	8.1	8.4	4.2
Female in Male Role	0.2	0.2	0.3	0.1	1.5
Female in Neutral Role	10.3	10.4	12.4	14.7	10.1
Sex-Role Mismatch	1.5	0.6	0.5	0.1	2.2

If it is yet further assumed that only problems about people are of any interest, and that the ratio of such problems to all problems is not related to the question of sexism, then Table 3 presents the clearest picture of textbook practices. Table 3 deals with the same information, but is restricted to considering only those problems about people.

Given the assumptions to make Table 3 the appropriate place to look, the period which clearly shows the least sex stereotyping is, happily, the L-70s. However, textbooks still have a long piece to go in order to be rid of sex stereotyping. Making yet another assumption, namely that the ratio of male role problems to female role problems shown by L-70s is acceptable, Table 4 shows how much change is still needed. Table 4 presents the "actual" and the "ideal" for the L-70s, the "ideal" being what should result from randomly assigning equal numbers of males and females to the roles given by the problems.

Table 3

Distribution of Story Problems for Five Periods for Grades Three through Six

(Entries are percentages based on problems about people)

	Period				
Grades 3-6	M-30s	M-50s	M-60s	E-70s	L-70s
About Children	89.1	88.4	78.0	74.2	63.0
About Adults	10.9	11.6	22.0	25.8	37.0
About Males Only	58.2	55.0	55.9	53.8	52.4
About Females Only	32.1	34.2	33.8	42.7	41.3
About Both or Either	9.7	10.7	10.3	3.6	6.3
Male in Male Role	21.1	22.0	22.1	28.3	27.8
Male in Female Role	2.0	0.8	0.3	0.0	1.7
Male in Neutral Role	35.1	32.2	33.5	25.4	22.8
Female in Female Role	15.8	15.1	13.2	15.5	10.9
Female in Male Role	0.3	0.3	0.5	0.2	4.0
Female in Neutral Role	16.0	18.8	20.1	27.0	26.3
Sex-Role Mismatch	2.3	1.1	0.8	0.2	5.8

Table 4

Actual and Ideal Assignments of Persons to Roles for Grades 3-6 Based on the Three Series with Widest Usage in Iowa in the Late '70s

(Entries are percentages based on problems about males only and about females only)

	Actual	Ideal
About Males Only	55.9	50.0
About Females Only	44.1	50.0
Male in Male Role	29.7	17.0
Male in Female Role	1.8	6.8
Male in Neutral Role	24.3	26.2
Female in Female Role	11.7	6.8
Female in Male Role	4.3	17.0
Female in Neutral Role	28.1	26.2

Given all the assumptions to make Table 4 the appropriate place to look, there clearly remains a reluctance to place males in female roles or females in male roles. This may be much more a matter of concern than the fact that textbooks have not generally presented equal numbers of males and females, since this protecting of role-boundaries is the kernel of the sex-stereotyping problem.

Of the three strategies mentioned in the first paragraph of this paper for eliminating sex-bias from textbooks, the choices made are fairly clear. Heavily used is the first choice, removing people. Hesitantly approached to a very modest degree is the third, the random assignment of equal numbers of males and females to roles.

The second strategy mentioned, while not much used by textbooks in children's hands today, deserves attention here because a number of publishers and leaders of the movement against sexism are strongly suggesting its use. Is the use of gender-free labels a wise choice? The following experiment (study) is reported here to suggest that the practice of using gender-free labels may be considerably less than desirable.

Four verbal problems were presented to each of 20 pre-school children, 20 fourth graders and 20 eighth graders. In each age group there were 10 boys and 10 girls. Problems were presented to one child at a time. Each problem was read to the child with a subsequent request that the child respond to a what-should-the-person-do question or to a how-should-the-person-do-it question. All problems used gender-free labels. Each child received two problems involving persons in male roles and two problems involving persons in female roles. The following sample problems are accompanied by information on which age group received the problem and on which role (M or F) was assigned to the problem.

1. A tree-surgeon is a person who cuts sick branches out of trees. A tree-surgeon is very high in a tree. Suddenly a swarm of angry bees begins flying toward the tree. What should the tree-surgeon do? (pre-school, M)

2. A secretary can type a page in five minutes. The secretary wants to know how much time is needed to type eight pages. How should the secretary find the answer? (4th, F)

3. A beginning sports announcer made 12 grammatical errors on Monday, 10 on Tuesday and 2 on Wednesday. To find the average number of errors made per day, what should the sports announcer do? (8th, M)

4. A school librarian noticed 12 books were stolen on Monday, 10 on Tuesday and 2 on Wednesday. To find the average number of books stolen per day, what should the librarian do? (8th, F)

Most children began each response with "he should..." or "she should..." A response was classified as a "gender-role match" in the case of a consistent use of third person masculine with a male role or of third person feminine with a female role. A response was classified as a "gender-role mismatch" in case of a consistent use of third person feminine with a male role or of third person masculine with a female role. A response was classified as "vague" if no pronouns were used, plural pronouns were used, a mix of pronouns not in agreement was used, etc. Table 5 shows the results.

Table 5

Assignments of Gender to Role for Problems Using Gender-Free Labels
(Entries are percentages)

	Gender-Role Match	Gender-Role Mismatch	Response Vague
Pre-School	71	9	20
4th Grade	81	6	13
8th Grade	79	2	19

Of special significance is the fact that all responses giving a gender-role mismatch were gained from problems involving the roles doctor, bus driver and coach, all roles for which the community had enough highly visible models to both make the community atypical and call into question the investigator's assignment of roles to these occupations. Thus, gender-role mismatches were observed only for roles which were close to being neutral roles on the basis of who held them in the community. It is therefore hypothesized that the use of gender-free labels tends to result in the child's assigning genders to roles in a way consistent with what the child's community exhibits. Furthermore, it seems reasonable to assume that an association is strengthened each time the child assigns the same gender to a role. If so, the use of gender-free labels tends to preserve whatever sex stereotyping exists in the child's community. Briefly, the use of gender-free labels at best avoids the issue and at worst contributes to the problem.

In summary, the positions taken by the writers are the following:

1. Sexism in textbooks prevails, now mainly in the form of a reluctance to place males in female roles and females in male roles.
2. Removing people from textbooks, which has been widely done, simply avoids the problem.

3. The use of gender-free labels at very best avoids the problem and may contribute to the problem of sex stereotyping.

4. The way to deal with the problem is to use half males and half females randomly (or at least nearly so) assigned to roles. Doing so just might make a difference over the ten or more years a child studies mathematics. We'll not know without trying, and we haven't tried yet.

MATHOPHOBIA: A MINI-COURSE FOR THE MATHOPHOBIC

Ruth Afflack

Sex-role stereotyping in mathematics has been influential in detering women from entering the field of mathematics. This is illustrated by the covers of two versions of the commercial game Battleship. This game is played on a grid representing the Cartesian coordinate system. In the original version of Battleship, the cover showed a father and son playing the game while in the background the mother and daughter cheerfully did the dishes and observed the play. The more recent cover shows only a girl and boy playing Battleship. With the earlier role model it is not surprising that women avoid mathematics.

As women begin to enter non-traditional fields, those who previously avoided mathematics, considering it unnecessary for their potential careers, now realize the necessity for mathematics in their careers. Frequently the avoidance of mathematics in high school, when mathematics becomes optional, causes women to feel mathematically incompetent or actually fear mathematics. Presently courses and programs are being developed to combat this insecurity in mathematics, particularly among women. One such course, Mathophobia, is taught as an intensive weekend offering through the Extension Program at California State University, Long Beach.

Women and Mathematics, the Problem and Effect

For years women have entered "traditionally female" occupations such as education, clerical work and the helping professions. Even as women became aware of possible admittance into other non-traditional fields, they were still entering the traditional fields in college. In 1972 sociologist Lucy Sells surveyed the entering University of California, Berkeley first-year students and discovered that 57 percent of the men as opposed to only 8 percent of the women had taken the necessary high school mathematics required to major in areas outside of education and the helping professions.[1]

By avoiding high school mathematics, women have limited their educational opportunities and career options. As society becomes more technological, and computers infiltrate every aspect of life, women who lack confidence with mathematics are barred from advancement or prevented from entering the more lucrative fields. A number of women who have written me concerning their perceived mathematical inability and accompanying fear of mathematics are completing advanced work in other disciplines; yet they still feel they cannot learn mathematics. Remedial review is not what is needed for these women, rather an opportunity for them to explore mathematics as a creative discipline and overcome their anxiety.

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[1] L. W. Sells, "Critical Points for Affirmative Action," in Toward Affirmative Action #3 (Autumn 1974). New Directions for Institutional Research, edited by L. W. Sells, Jossey-Bass, San Francisco, California.

Women who are trying to advance in the business world are confronted with obstacles related to mathematics. One mathophobic student from the television industry who was a strong supporter of women in her profession, lacked the confidence in her own mathematical ability. Simple mathematical problems such as calculations involving percents appeared to block her ability to consider the overall project and her advancement depended upon competency with budgets. Again math anxiety prevents this type of woman from achieving her goals.

As women become more aware of career options in mathematics, science and engineering, they realize the importance of a solid foundation in mathematics. With the recent trend toward a more technological society, many academic disciplines are requiring more mathematics. Computers are being used in such areas as History and English. Libraries are being reorganized through computers. Social and behavioral science fields are incorporating statistics as prerequisites for their courses. A faculty member applying for a grant through the California State University and College system must complete a computer search prior to submitting a proposal. It is becoming more difficult for persons uncomfortable with mathematics to survive in society today.

Women re-entering college are finding their lack of mathematics preparation a handicap in their new college careers. The psychological effect on these women is devastating when they confront mathematics without confidence. A woman in one of my classes confessed that members or her family, who were employed or were students in engineering, had little respect for her because she "couldn't do" mathematics.

At my university several women faculty have privately expressed interest in my course while indicating they probably could not be helped. One friend has jokingly suggested I accompany her to a family dinner so that I could argue with her brothers in engineering and business who claim she is not logical. This woman is a professor of English. In teaching composition she shows her students how to clearly organize ideas and defend a thesis; still she is convinced she cannot logically defend her position in the family debates.

Today women recognize the need for mathematics. That boys, not girls, are expected to take mathematics and do well is a situation which may be changing. But women who lack a mathematical background, whether because of societal or peer pressures, negative attitudes and experience, previous avoidance or an induced feeling of mathematical inadequacy, are demanding an opportunity to overcome their math anxieties. In response to the article on "Math Anxiety" by Sheila Tobias in Ms. Magazine,[2] I have received over 300 requests for help from anxious women and from educational institutions. It is the responsibility of mathematics educators to provide solutions for these women.

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[2] Sheila Tobias,"Math Anxiety," Ms. Magazine (September, 1976).

Development of the Mathophobia Course

The ideas for my course Mathophobia evolved over a period of years, beginning with my participation in the SEED (Special Elementary Education for the Disadvantaged) Project. Under the direction of Bill Johntz, this program placed mathematicians in elementary classrooms for short periods daily to motivate students by using the discovery method of teaching mathematics. The curriculum was open-ended, dependent primarily upon the teacher. This was my introduction to the inquiry approach to teaching mathematics and to the use of mathematical games and activities in the classroom.

Simultaneously, while participating in SEED, I was teaching a course on mathematics for elementary teachers at California State University, Long Beach in a traditional manner. I realized I was not reaching the prospective teachers in my class at CSULB but I was turning on the elementary school children in Watts. Therefore, I began to alter my University course to include mathematical materials and activities which challenged the prospective teachers while giving them useful models for their future teaching.

As I continued to develop new ideas, my repertoire expanded until I had enough material for a separate one-unit course for mathematics teachers, elementary and secondary. When mathematics graduate students and faculty participated in the first offerings of the course, I realized the potential use of this type of mathematical activity.

During a discussion of possible mini-course offerings in Women's Studies, I responded positively to an inquiry about a mathematics course for women who had avoided taking mathematics courses. These women who now needed mathematics for requirements in their major had developed a fear of mathematics. Responding to these needs, I felt I might incorporate some of my mathematics projects into a course which would encourage college students, particularly women, to take a mathematics course. The first offering was titled "Math for Ms," which was later changed (not at my suggestion) to "All you wanted to know about mathematics but were afraid to ask for fear of failing." Since the later title sounded too encompassing, I renamed the course "Mathophobia" to attract the attention of the intended student.

Although Mathophobia is not a proper designation, when accompanied with the course description, it conveys the intent of the course, which is to help students overcome their fear of mathematics, develop their mathematical competency and appreciate the challenge of the subject matter.

Contributions from Personal and Professional Experience

When constructing the Mathophobia course I reflected on my own reasons for choosing mathematics as a career. First, my father was a mathematician. Our family entertainment frequently consisted of mathematical games. My confidence in mathematics had developed through an understanding and appreciation of the subject before I entered school, even though I do not recall ever being taught mathematics at home. My positive attitude resulted from the family activities, in which mathematics had been fun. Interestingly, my brother changed his major during his freshman year from engineering to music, my mother's specialty, while my sister and I pursued mathematics.

From my experience with encounter groups during the Vietnam era, I learned that successful groups were organized around a constructive project. Support alone, without cooperative activity, did not bind the group nor sustain the members outside the group. This influenced my thinking about the mathophobic students: why not create a supportive environment in which students would be involved in doing mathematics cooperatively?

As a teacher, my background had also provided insight for teaching mathophobics. After participating in Project SEED I had changed my methods of teaching mathematics. There were no wrong answers in the SEED classes. Games and activities were employed to teach basic concepts of mathematics. Much of the success of the program had come through an improved self-image. One particular example comes to mind. I'll never forget Edward, a second grader, who had not had positive recognition in his class. When playing a simple game using paths on a grid of numbers, Edward was able to point out the answer even though he had difficulty writing his name or numbers. After the game was renamed "Edward's game" and Edward became the teacher, Edward changed dramatically, his hand would be raised almost as soon as I entered the room.

I had extended the use of games and mathematical activities to the teaching of mathematics for elementary teachers. These students often disliked or were afraid of mathematics. With negative attitudes toward mathematics, they avoided taking more than the required classes in mathematics and procrastinated in taking those classes. These students could be quite specific about the onset of the mental block in mathematics. One such student told me that her inability to do mathematics dated back to the day in fifth grade when her teacher, who had on a red dress, hit her with a ruler.

The use of games with the prospective teachers not only helped to break down their apprehension about mathematics by showing them that mathematics could be fun, but provided them with material applicable to their teaching. As future teachers, it was important that they have positive experiences with mathematics so that they would not carry their insecurity and dislike into their classrooms. Also, once the resistance to mathematics is overcome, they gain confidence in the subject matter and enjoy mathematics. My primary goal was to change their attitudes so they could learn mathematics. This certainly was applicable to mathophobics.

Our University as part of the California State University and College System had implemented an EOP (Educational Opportunity Program) which allowed a percentage of students, usually minorities, to enter college without the necessary entrance requirements. Similar to the results reported in the Sells study of women at Berkeley, these EOP students frequently do not have the proper mathematics background to be eligible to take a beginning intermediate algebra course. Without algebra EOP students were excluded from many academic areas. Therefore, I developed a special algebra section, primarily for EOP or returning women students. Besides being understanding of the mathematical backgrounds of these students, I was spending class time encouraging them to overcome their "I can't do it" syndrome. Their self-image improved. In class, "doing" mathematics was stressed with individualized and peer group assistance. They learned to read mathematics. The cooperation—instead of competition—among the students in the class makes the class unique. It is usually the only class I teach in which students do not leave the room until the next class is about to begin.

With this background in teaching students who lacked confidence and a proper background and who disliked or feared mathematics, I was willing to attempt a class for Mathophobics. The focus in the mathophobia class would be on "doing" enrichment mathematics rather than dwelling on the students' weaknesses or teaching remedial mathematics. If these students could overcome their anxieties, then they could take the necessary mathematics courses.

Course Description

Mathophobia is a course for students who feel insecure with mathematics, either because they have avoided the subject or have built up a fear of mathematics. Through the use of games, puzzles, and mathematical activities, a variety of topics are introduced which show that mathematics can be fun as well as challenging. After identifying and analyzing a given problem, students apply problem-solving techniques and then generalize the relevant mathematical concepts. Mathematics is learned by "doing." To overcome their fears and gain mathematical confidence, students work in a supportive environment either individually, as a class, or in small groups.

Philosophy of Course

Although the course is titled Mathophobia, the purpose is to help students alleviate their fears of mathematics. Primarily the goal is to expose students to a variety of topics in mathematics in a supportive environment. Through the use of games and activities students feel more comfortable with mathematics. In analyzing the puzzles or problems, logical reasoning and problem-solving strategies are introduced. Given a problem or game, students forget that they cannot "do" mathematics. The involvement in the activity along with support from other students with similar mathematical deficiencies helps students recall their math and gain confidence.

A supportive environment is crucial in the teaching of this class. Beginning with the initial discussion of why they are taking this class, students are encouraged to work together, talk to each other during class, and ask questions at any time. Students must be encouraged to admit their weaknesses. If they don't know how to add fractions for instance, they may interrupt the class or ask their classmates or an instructor for help. When students feel uncomfortable, they may take a break (go for a walk) or go to an area in the room where there are books, games and activities. Students should feel free to leave a threatening situation whenever necessary. It is the openness of the classroom, the active participation and support of other students which helps alleviate the fears.

Mathematics is not for spectators: even visitors in the Mathophobia class participate. Learning mathematics requires active involvement, not passive listening. Especially in this class, students are encouraged to question and "do" mathematics.

Pattern recognition is one emphasis in the class. Students are frequently asked to "look for the pattern." So often in mathematics classes, after examples are given, students are asked to solve analogous problems by following a generalized pattern.

Guessing is encouraged. Often students are afraid to take the initial step in solving a problem. If they can at least guess an answer, there is a starting point. Also, a guess cannot be wrong, and it helps the student get a feeling for the problem while taking the first step.

Logical reasoning is fundamental to most mathematical problem solving. Logic games are played where students must verbalize their reasoning. Throughout the course, the reasons are asked for—which allows the students to recognize similar reasoning patterns.

Problem solving is seldom taught. Mathophobic students in particular often feel that mathematicians somehow know the correct formulas and can write solutions directly. Mathematicians must have magical "insight," as one student wrote. Problem-solving strategies are discussed throughout the course and not covered just as a special topic, as is frequently done in high school algebra in the unit on word problems. It is exciting when students try a problem-solving technique on a new type of problem and it works.

A short take-home problem set is given, after a similar practice set is gone over during the last class session. Since one of the objectives of the course is to build the student's confidence, the take-home assignment on which they can get help gives them the opportunity to try mathematics on their own. Students get positive feedback when they show the problems to someone else, since they at least have an understanding of what the problems require for a solution. Knowing they can attack a problem by themselves, even if they don't find the best solution, is a positive experience for most Mathophobic students.

Class Structure

The course consists of a variety of topics in mathematics, each introduced by a game, activity or puzzle. In the ideal situation, a short quiz is given at the conclusion of each topic. Students may work together or receive help on these exercises, which reinforce the basic concepts while providing feedback of the students' progress and exposing any weaknesses in their mathematical abilities.

Since the backgrounds of the students are quite diverse, as are their expectations of the course, it is made clear at the outset that the course will not include a thorough presentation of any one particular field of mathematics. The exposure to a wide variety of mathematical ideas allows the emphasis to be placed on "doing mathematics."

In summary, the class is conducted as openly as possible. Students are encouraged to ask questions and discuss their ideas with other students. Materials and games are available in the room for students to explore. Much of the activity involves hands-on materials which the students share in small groups. If students' questions indicate a need or desire to review certain aspects of mathematics, such as division of fractions, either the material will be covered by the class or an assistant will work with a small group of students.

At the beginning of the first class meeting, those present (including faculty and student assistants) introduce themselves and tell why they are in a mathophobia class. The enrolled students are asked to indicate their mathematical backgrounds and any specific mathematical needs, including courses required for their major.

The students' fears are not dwelt on during class: the emphasis is on creative mathematics. If during the session a student's anxiety builds up over a specific topic, that person knows there are options: a person can share her/his feelings with the class, can feel free to take a break, can ask for individual help or can join with others in working on the problem.

Guessing is encouraged since it is critical in this class. Often adults tend to be intimidated; they are afraid to let go, make a guess, or try an unfamiliar approach. Function machines assist in encouraging guessing since the answer cannot be wrong until the "rule" of the machine is determined (see Example 2a in Appendix B).

Students are requested not to call out answers, so that all students have an opportunity to "discover" the solution. Also specific answers are frequently not provided, rather hints are given or similar problems explained. This procedure helps students to persevere to find the solution on their own or with a group.

By learning problem-solving techniques, much of mathematics can be demystified. For instance, students recognize that mathematicians also use trial and error in solving problems and do not merely produce results with mathematical insight. One student, on a take-home exercise, kept on trying to solve a pouring type problem; she didn't give up. After 21 steps, she found a solution. Elated she called her husband at work, then went back and reviewed her work to determine where she could eliminate steps to produce the desired seven minimum steps. For her this was a real breakthrough.

Besides the topic quizzes, there are class projects, numerous optional handouts for individual or group work and two take-home problem sets, one a practice set. Students have been asked to write a paragraph on their feelings, whether positive or negative, about some incident or topic in the class about which they felt elated or frustrated.

The enthusiasm in the class is contagious. This response comes from the cooperation between students, assistants and faculty. The common factor which binds the class together is the insecurity with mathematics and the creative energy generated from constructive involvement. As frustrations rise, they are frequently overcome through mutual support and willing assistance. Students deal first with their own mathematical problems, then when they understand a concept, they in turn help others. As one student said, "There was a common denominator in the fear and panic most of us felt about math in varying degrees. Maybe that replaced a feeling of competition with a feeling of cooperation —sort of a group spirit."

The best way for me to share with you the effect of the Mathophobia course is through the students' own words. Here are some direct quotations:

> When it was over, I realized that along with the right answers, I was understanding concepts. It wasn't just mechanical.
>
> I never realized before that one could look at patterns. In this respect math is no different than any other discipline; e.g., psychology, philosophy, etc. It's a way of looking at problems, not just those involving numbers. This was a revelation to me.
>
> I found I was enjoying playing with the problems, the way I enjoy reading. I sat for what turned out to be three hours last week working on the logic puzzles—I checked the time and I was amazed.
>
> Also forming the geometric figures was an eye opener. I had never grasped the concept of a 3-dimensional figure until I made them out of the cardboard.
>
> Although I felt very frustrated in your "Mathophobia" class a good deal of the time, I did realize that you were not going to give me the answer. Faced with the fact that I had to do this myself, I resorted to figuring out as many problems as I could on my own. [On this person's take-home problem set, I found the following: "I did it! I'm so excited—no one helped me—I sat down, concentrated and did it!!"]
>
> I liked the whole approach of this class. It helped me to see the creative side of math, rather than always seeing it as a jungle of formulas and numbers.
>
> There was a great sense of accomplishment when the mental bell rang and I actually figured out two function machines by myself.

I always thought I was too dense to figure something like the function machines out—BUT I'M NOT DENSE and I can do it.

I have always felt that Math is hard and boring, and my statistics classes have given me fits, so I came to this class for help with no thought of enjoyment. I found myself enjoying two Monday night classes after beginning work at 6:30 A.M. and giving up a full Saturday and telling everyone that this was really a neat class.

Math has always been an enigma to me and most of it still is. I still have difficulty setting up any type of math problem, let alone solving it. But thanks to you, I can now now see it can be tackled in a step-by-step fashion.

Ever since third grade I have been unable to do multiplication. Even remedial math didn't help. Then you explained the peasant method—what a thrill for me to finally be able to do a multiplication problem, no matter how slowly, and have it come out right every time.

I feel that the most positive idea I got from the class is that a lot of math success is through trial and error and not giving up. I've sometimes had the feeling that people who are good at math have some type of "magical" insight into problem solving that I lack.

I think I finally see—because I touched the geoboards—what the area of a triangle is all about.

There was a common denominator in the fear and panic most of us felt about math in varying degrees. Maybe that replaced a feeling of competition with a feeling of cooperation—sort of a group spirit.

I still have much to learn—and relearn—but the ability to approach new problems as intriguing puzzles to be figured out will produce results that the paralyzing fear I used to experience never could. I enjoyed the class and for the first time in my life I think math is an enjoyable challenge.

APPENDIX A

COURSE CONTENT

This outline, by topics, contains sample games, activities and materials used in the Mathophobia class. Due to the time constraints of a weekend course and the particular needs of a given class, all of the materials listed are not covered in each offering of the course.

A. LOGIC

Listing Logic as a topic is not meant to imply that logic should be considered as a separate topic only. Logical concepts and reasoning are integrated throughout the course. Inference patterns followed by arguments pervade throughout the sessions. However, specific examples of conjunction, disjunction, and negation are mentioned along with the concept of valid statements and conditional statements. Counter-examples are sought; necessary and sufficient conditions are examined; and the basic principle of hypothesis-conclusion is described and incorporated.

1. Games

    a. Mastermind
    b. Martin Gardner's Paradox Box
    c. "Dienes" type attribute games
    d. Twenty questions
    e. Crypt-arithmetic
    f. Wff-n-proof

2. Activities

    a. Logic puzzles having 2-5 categories, each with several members. In these puzzles, a set of clues is given, which when correctly analyzed has a unique solution (see Appendix A-1a).

    b. Logic puzzles consisting of a series of statements, only one of which, for instance, is true (see Appendix A-1b).

    c. Counterfeit coin problems using a given number of weighings on a balance scale.

    d. Pouring problems in which, perhaps, a full container must be evenly divided, using empty containers with prescribed volumes.

    e. Problems in which a certain class of people always lie while another class always tell the truth.

B. PROBLEM SOLVING

Again problem solving should not be considered as an entity by itself. Problem-solving strategies and techniques are applicable to all topics and activities; this listing merely indicates certain activities which help illustrate problem-solving methods.

1. Games
    a. Clue
    b. Think-a-Dot
    c. Matchstick puzzles
    d. Coin puzzles
    e. Nim type games

2. Activities
    a. Puzzles, such as Tangram
    b. O'Henry type story problems
    c. Tricks, especially spatially related
    d. Set breaking activities

C. FUNCTION MACHINES

Function machines are used whenever possible in the course to help students organize data, recognize patterns and generalize concepts. They provide an inductive approach to reasoning and benefit students in understanding relationships. Function machines motivate students to guess answers, since without sufficient information many answers are possible. For example, $n^2 + n + 11$ is not a representation of a prime number for all counting numbers—let $n = 10$. Examples of the basic function machine are provided in Appendix A-2.

1. Games and puzzles
    a. Least moves games, such as invert a coin triangle
    b. Brahman Tower Puzzle
    c. Eight person jump
    d. An n x n x n cube painted on the outside; identify the number of smaller cubes painted on 0,1,2, or 3 faces

2. Types of function machines
    a. Figurate numbers
    b. Euler's formula for polyhedra
    c. Pick's Theorem
    d. Number of diagonals of an n-sided polygon
    e. The handshake problem, an application
    f. Fibonacci steps and paths
    g. Name, words, symbols
    h. Minimal number of regions, points, intersections
    i. String/paper cutting and folding

D. GEOBOARDS

The geoboard, a valuable "hands-on" mathematical aid, is a square board with a lattice of pegs on which rubber bands are stretched. Students enjoy using the geoboards while formulating concepts of measure. The idea of "measuring unit" is thoroughly discussed and applied. Formulas are "discovered;" laws are illustrated; and definitions are clarified. Pick's Theorem using the boards is an excellent function machine application. Students are startled when asked to construct a square with an area of 5 square units. An example of geoboards is included in Appendix B-3.

1. Geoboard Activities
    a. Guess my figure in 20 questions
    b. Count paths
    c. Geometric construction problems

2. Concepts
    a. Unit, measurement
    b. Triangles: type, area, perimeters
    c. Polygons
    d. Algebra formulas, such as $(3a + 2b)^2$
    e. Pythagorean theorem
    f. Multiplication, distributive law, fractions
    g. Translations and rotational symmetry

E. COUNTING

The fundamental Principle of Counting is explained and illustrated using tree diagrams. Systematic approaches are devised for counting in the various activities.

1. Games and activities
    a. Mastermind-codes
    b. Card tricks
    c. Magic squares, cubes
    d. Polyominoes
    e. Abacus
    f. Kalah

2. Exercises and concepts
    a. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
    b. Order relations
    c. "How many" problems
    d. Permutations, combinations with/without repetition
    e. Word-symbol function machines
    f. Figurate numbers

F. ARITHMETIC - NUMBER THEORY

Often students request a review of certain arithmetic operations, such as fractions. Other algorithms are presented which intrigue students while helping to explain our basic operations.

1. Games and activities
    a. Tricks using calculations, such as dividing a 6-digit number, abcabc, successively by 7, 11, and 13 to obtain abc
    b. Decoding puzzles, see Appendix A-4
    c. Prime/Factor game derived from an article in Scientific American by Martin Gardner

- d. Cross number puzzles
- e. Writing numbers from 1 to 100 using arithmetic operations on a given set of integers
- f. Complete the box puzzles
- g. Numo
- h. Numble

2. Concepts

- a. Equal addition method of subtraction
- b. Duplication/mediation method of multiplying
- c. Napier's rods
- d. Cuisenaire rods
- e. Operations with fractions
- f. Calculating devices
- g. Even-oddness and primes
- h. Number patterns

G. ALGEBRA

Students appreciate simple word problems which can be analyzed. Abstract notation is introduced both in the activities and explanations.

1. Games and activities

- a. Trick word problems
- b. Calendar problems
- c. Pick a number type calculation puzzles, see Appendix A-5
- d. Equations

2. Concepts

- a. Symbolism
- b. Equations, especially of the type $ax + b = c$
- c. True interest rate
- d. Polynomials
- e. Exponents and powers
- f. Inequalities

H. GEOMETRY

For some students one of the most exciting sessions is constructing three-dimensional models or polyhedra, without being given a pattern or instructions. Spatial relations are emphasized since this area is often weak for Mathophobics. Rarely are students given definitions; rather they are led in well-defining terms until a particular definition is precise.

1. Games and activities

- a. Cubic, Clusters and Chains and Space Checkers
- b. Mind Maze
- c. Psyche Paths

d. Geodesic dome
e. Model construction and nets
f. Tetrahedron puzzle
g. Battleship-Cartesian Coordinates
h. Polyominoes and Soma Cube
i. Tangrams
j. Mirror cards

2. Concepts

a. Polygons and polyhedra definitions, nets, area, volume
b. Tessellations
c. Pythagorean Theorem
d. Spatial concepts
e. String designs
f. Plane drawings
g. Plane geometry; congruence, similarity, symmetries
h.

I. NUMERATION SYSTEMS

Students gain an understanding and appreciation of our number system through word puzzles (see Appendix A-5) and activities in other numeration systems. They further develop insight into abstractions and learn to use symbols and recognize correspondences. This area of mathematics is not overly stressed however.

1. Games and activities

a. Dr. Nim and Nim
b. Tower puzzle
c. Decisions
d. Games using other bases or numeration systems such as Buzz or Roman numeral match stick puzzles
e. Guess my number

2. Concepts

a. Explanation of games and activities which use the binary number system, such as "peasant multiplication"
b. Weighing and coin problems with limited weights
c. Other numeration systems
d. Place value
e. Modular systems

J. TOPOLOGY

Another exciting session involves topological activities. Again students appreciate the emphasis on spatial relations and visualization. Recognizing topological equivalences is a new concept for many students.

1. Games and activities

a. Mobius strip
b. Topological rope tricks
c. Four color game
d. Network puzzles

2. Concepts

   a. Topological equivalences, using variations of simple puzzles
   b. Network theory including "Königsburg bridge problem"
   c. Four color problem
   d. Definitions, such as simple, closed curve

K. MISCELLANEOUS TOPICS

1. Art

   a. The works of Escher, Vasareley and Max Bill
   b. Coloring designs—Escher, Altair or Op Art
   c. Tessellations
   d. Mathematical illusions
   e. Stereograms

2. Statistics and Probability

   a. Hex-stat demonstrator
   b. Misuse of statistics
   c. Probability of games of chance
   d. Averages
   e. Pascal's triangle

3. Computing devices

   a. Calculators
   b. Computers
   c. Abacus
   d. Slide rules and nomographs

4. Metric

   a. Basic units
   b. Conversions

5. Consumer

   a. True interest rate
   b. Banking
   c. Specific topics

6. Special topics

   a. Fibonacci numbers
   b. Magic squares, cubes

APPENDIX B

SAMPLE ACTIVITIES

A. LOGIC

1. There were five women entered in the women's 100 meter freestyle at the Olympic tryouts in Long Beach: Sheila Jones, Maria Aguilla, Eve Toshihama, Enid Schmelter, and Joanne Finley. They represented CSULB, UCSB, SDSU, UCLA, and SMCC, not necessarily in that order. Can you determine how each woman finished the race, in which lane she swam and what school she represented?

   a. No woman's finishing position was the same as her lane number.
   b. Long Beach was represented in lane 4.
   c. Although behind on the first turn, Enid came in ahead of the swimmer in lane 1 and the swimmers from Santa Barbara and Los Angeles.
   d. The Los Angeles swimmer had just recovered from the Swine flu, so was not at her best as she came in last.
   e. The swimmer in lane 3 decided not to go to school in her hometown, Santa Barbara, although she really liked her coach there.
   f. The swimmer in lane 5 finished third.
   g. Jane Finley seemed to slip as she left the starting block, but she recovered the lost time and didn't finish last.
   h. The swimmer in lane 2 was thinking of transferring to San Diego or UCLA.
   i. The swimmer in lane 3 did not qualify since she didn't finish in the first three places.
   j. Maria Aguilla, the youngest woman in the race, won the race.
   k. The San Diego swimmer's time was .10 seconds more than her competitor from Santa Barbara.
   l. The swimmer in lane 2 had been interviewed prior to the race with Sheila Jones and the Santa Barbara woman.
   m. Sheila Jones was majoring in marine biology at San Diego.

2. Each of the three Simon sisters was employed in a different profession, engineer, artist or doctor.

   If you know that only one of the following statements about the women and their professions is true, can you determine Ms. Simon's profession?

   Ms. Anderson is not an engineer.
   Ms. Williams is not a doctor.
   Ms. Anderson is a doctor.
   Ms. Williams is not an engineer.

B. FUNCTION MACHINES

For each machine find one rule which relates what goes "in" to the machine with what comes "out" of the machine, that is in illustration 2, how do you you get 14 using 5? Each machine has one rule.

1.

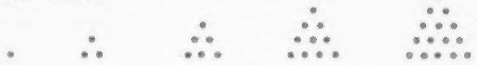
IN	OUT
Cathy	C
Robert	R
Denise	D
Melvin	M
Edward	D
Anne	N
Jane	—
Ruth	—
Ilene	—

2.

IN	OUT
1	2
2	5
3	8
4	11
5	14
⋮	⋮
9	26
⋮	⋮
100	—
⋮	⋮
n	—

3. A procedure to find the general rule

Triangular numbers:



number of dots on bottom row	total number of dots
1	1
2	3
3	6
4	10
5	-
6	-
7	-
⋮	⋮
n	-

IN	OUT
1	1 = 1 x 1
2	3
3	6 = 3 x 2
4	10
5	15 = 5 x 3
6	21
7	28 = 7 x 4
⋮	⋮
n	-

IN	OUT
1	1 x 1
2	2 x 1-1/2
3	3 x 2
4	4 x 2-1/2
5	5 x 3
6	6 x 3-1/2
7	7 x 4
⋮	⋮
n	n x ?

IN	OUT
1	1 x 2/2
2	2 x 3/2
3	3 x 4/2
4	4 x 5/2
5	5 x 6/2
6	6 x 7/2
7	7 x 8/2
⋮	⋮
n	$n \times \frac{n+1}{2}$

4. A machine with two 'in' entries

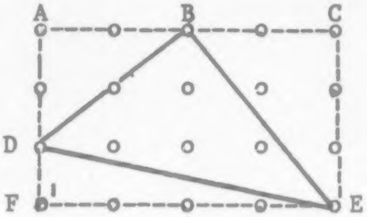
IN		OUT
2	3	7
3	5	11
5	2	12
6	8	20
10	13	33
⋮	⋮	⋮
a	b	–

5.

IN	OUT
R A G	■ ■
S I T	○ ○ ○
S A D	□
R I G	● ●
R U G	▲ ▲
S U D	△
S A T	□ □ □
S A G	______
____	●

C. GEOBOARD

The unit of measure is □ , which is 1 square unit. Then ▭ is 2 square units, ◹ is 1/2 square unit and ◹ is 1 square unit. To determine the area of triangle BED, first find the area of the rectangle ACEF, then subtract the areas of triangles ABD, BCE and DEF. We find the area of triangle ABD is 2 since it is one-half the area of square ABoD. Similarly the area of triangle BCE is 3 and the area of triangle DEF is 2. Calculating the area of triangle BED, we get [(12 - 2) - 3] - 2 = 5.



D. ARITHMETIC - DECIMAL NUMBER SYSTEM

Decode the word by substituting the correct digits (0,1,2,...,9) into the equations for the letters and then recording the letter representation of each digit into the corresponding blank. Each letter has a unique digit representation throughout the puzzle.

___	___	___	___	___	___	___	___	___	___
0	1	2	3	4	5	6	7	8	9

$R^2 = AR$

$\begin{array}{r} U \\ U\overline{)GD} \\ \underline{GD} \end{array}$

$A(L + L + L) = o$

$P \times P = P + P$

$A^Y = N$

E. ALGEBRA

Multiply the number of the month you were born in by 5. Add 13 and multiply the resulting sum by 10. Subtract 5 then multiply the difference by 2. Finally add the number of the day on which you were born. What do you have to do to the final result to determine your birth date?

Let M = the number of the month you were born in, and D = the number of your birth date. In the example, M = 5 and D = 23, that is, May 23 is your birthday.

	Example	Generalization
Multiply M by 5	5 x 5 = 25	$5M$
Add 13	25 + 13 = 38	$5M + 13$
Multiply by 10	38 x 10 = 380	$(5M + 13)10$
Subtract 5	380 - 5 = 375	$(5M + 13)10 - 5$
Multiply by 2	375 x 2 = 750	$\{(5M + 13)10 - 5\}2$
Add birthday	750 + 23 = 773	$\{(5M + 13)10 - 5\}2 + D$

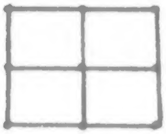
Simplifying the generalized expression, we get 100M + D + 250. Therefore we must subtract 250 to determine the birth date. 773 - 250 = 523, showing May 23.

F. PROBLEM SOLVING

1. MATCHSTICK PUZZLE

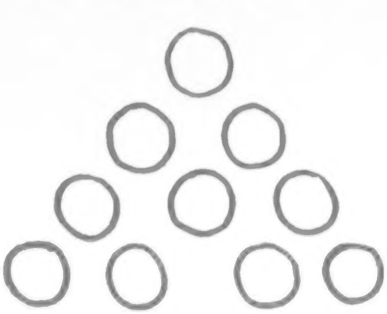
Using the 12 matchsticks in the grid as shown, try the following:

1. Move 2 matches and leave 7 squares.
2. Remove 2 matches and leave 2 squares.
3. Move 3 matches and leave 3 squares

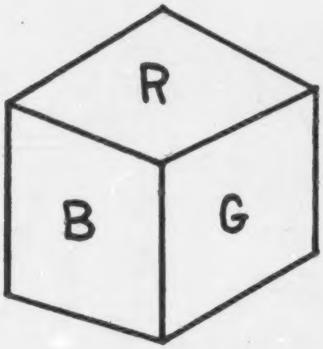


2. INVERT THE TRIANGLE

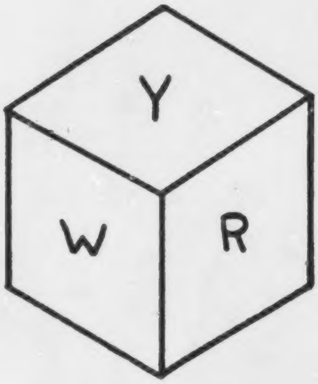
Form a triangle with ten pennies as shown. By moving only 3 coins, invert the triangle (make it point down instead of up).



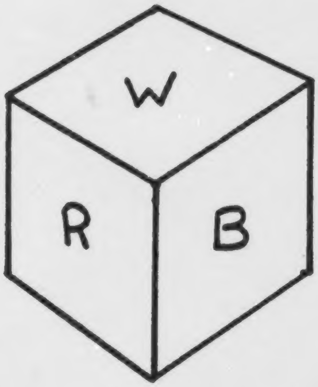
G. SPATIAL



View A



View B



View C

Three views are shown of a cube painted blue (B), green (G), red (R), white (W), and yellow (Y).

Can you determine the color of the face opposite the red face in View A?

H. ARITHMETIC

Decode

1. Write any three-digit number in which the first and last digits differ by more than 1.
2. Reverse the digits of your number to form another three-digit number.
3. Subtract the two numbers in steps 1 and 2 to obtain a positive number.
4. Reverse the digits of the number obtained in step 3.
5. Add the two numbers obtained in steps 3 and 4.
6. Multiply the sum from step 5 by one million.
7. Subtract 123,957,683.
8. Decode your answer by replacing each digit according to the code:

0	1	2	3	4	5	6	7	9
H	U	S	F	I	T	A	N	M

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PROGRAMS TO COMBAT MATH AVOIDANCE

Rita May Liff

Women and girls often place limits on themselves when it comes to dealing with mathematics. In high school few women choose to study math after it becomes optional. In college most women major in fields requiring little or no math. In the working world, nearly all the math-oriented fields are male-dominated.

To increase the involvement of women in mathematics, efforts must be made to help women feel more positive about math and their ability to do it, to destroy the notion that math is an inappropriate pursuit for females, and to increase awareness of the limitations a poor math background imposes on career options.

A variety of programs being carried out in the San Francisco Bay Area to increase women's involvement in mathematics will be described.[1] These programs represent efforts to tackle the problem of math avoidance at every age level, by working with children, young women, parents, counselors, teachers, college and resuming women students. Though the programs vary in duration, audience, and orientation, they share several key components. Each provides:

1. A SUPPORTIVE ENVIRONMENT—a place to openly discuss why women often hesitate to do math, and an encouraging atmosphere fostered by a conviction that women can do math if they choose.

2. VALIDATING EXPERIENCES—an opportunity for women to become involved in doing mathematics and to experience success.

3. AN EMPHASIS ON CAREERS—a focus on the idea of math as a "critical filter" in the job market.[2] This creates an awareness of the increasing number of jobs requiring mathematics, and the excellent opportunities for women today.

4. ROLE MODELS—a shattering of the myth that women "aren't mathematically oriented" by introducing students to women who have strong math backgrounds and who are enjoying success in traditionally male fields.

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[1] The author of this paper has had the pleasure of being involved in planning and implementing each of the programs described here. A significant number of other related projects are also underway in the S.F. Bay Area. See: Jane M. Day, "A Collection of Reports on Current Programs in Northern California Colleges to Combat Math Avoidance," College of Notre Dame, Belmont, California, January, 1978.

[2] Lucy W. Sells, "The Forum: Mathematics-a Critical Filter," The Science Teacher, Vol. 45, No. 2, February, 1978.

5. CONTINUING SUPPORT—a chance to form professional contacts and to gather information about continuing education and training.

MATH FOR GIRLS

This course has been on-going since 1974 and serves 6-14 year-old girls. It's offered once a week in eight-week sessions on a tuition basis at the Lawrence Hall of Science at the University of California in Berkeley—a public science center and museum. The course was developed by Dr. Diane Resek, who now directs the "Math Without Fear Program" at San Francisco State University.

The class is taught by female science and math students at the University of California who enjoy math and want to share their enthusiasm. The emphasis is on showing girls that they can do math, that math is fun, and that they will limit their future options if they drop out of math.

Building Mathematical Confidence and Competence

An enthusiasm for math is created by actively involving students in games, puzzles and logic problems which are challenging and interesting, but still assuring them of some level of success. Problems are selected which can be solved using many different approaches, and which don't depend on specific math skills that some students may lack. Problem-solving strategies are emphasized; these strategies include:

1. TRIAL AND ERROR: GUESSING -

   Many, if not most, students view success in math as depending almost entirely upon memorization; if they don't know a "formula" they won't even attempt to do a problem. In this course an environment is created in which students are encouraged to make guesses, to call out answers, and in which no one is allowed to call another's answers "stupid" or "wrong." Each suggestion or guess is seen as an hypothesis, contributing information which helps in narrowing down the possibilities. Wild ideas are encouraged, problems with unusual solutions are presented, to show that doing mathematics and solving problems involves taking risks, taking a change at being wrong. It is important that students view math in this broadened sense, as a way of thinking and a process for solving problems, rather than as a body of facts and rules to memorize. The "aha" feeling, the joy of discovery, comes about only when one is willing to try and to fail on the path to success.

2. METHODS OF RECORDING AND KEEPING TRACK OF RELEVANT INFORMATION -

   Successful problem solving involves careful consideration of many factors at once: What is given in the problem? What is one trying to figure out? What can be deduced from previous attempts at solving the problem? Many students benefit from being shown different ways of representing this information.

In solving word problems, for example, just having students write down what's given, and what's to be found, proves very helpful. Otherwise they often stare at a problem after one reading and claim they don't know where to begin. Restatement of a problem often suggests methods for attacking it. Many models are useful for recording data, such as diagrams, pictures of a problem, tables of information. Students are encouraged to use these models and invent their own.

3. SIMPLIFY/START AT THE END/BREAK THE PROBLEM INTO PARTS -

Though these strategies are obvious to experienced problem solvers, they must be pointed out to many students.[3] Whenever applicable, these strategies are recommended. Students might be told that when taking multiple-choice tests, for example, a consideration of the solutions given may provide a clue as to how to approach the problem. And of course many problems can be reduced or related to simpler problems which students can more easily solve, and from which generalizations can be made.

Certain classroom characteristics and techniques facilitate the risk-taking behavior one hopes to foster: exploratory, discovery-oriented problems; not allowing students to 'put down' others' ideas or to shout out "this is so easy" once they've figured it out; brainstorming techniques;[4] having students work in small groups and share ideas and methods.

Expanding Career Awareness

One of the reasons Math for Girls was originally offered was to increase the percentage of females attending Lawrence Hall classes (a survey revealed that only one-fourth of the students attending courses were girls). So part of the course is set aside for discussing sex-role stereotyping, and encouraging the girls not to limit their options because of it. One of the ways to initiate discussion is having students brainstorm a list of what they consider to be "male" and "female" careers. The lists are likely to be very different! The discussion might center around why they consider certain careers to be sex-typed, what role math could play in these careers, what their own aspirations are, etc. Typical responses to personal plans include being: a teacher, a nurse, a model, a keypunch operator. It can then be suggested that they also consider becoming a doctor, or a computer, programmer or an engineer. They may respond by saying that women just don't or even can't do these things. This is an ideal opportunity for the instructors to discuss their own choices and what they are doing. These role models provide convincing evidence that women can be anything they want to be.

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[3] A classic book on problem-solving strategies and heuristics is: G. Polya, "How to Solve It, A New Aspect of Mathematical Method," Princeton University Press, Princeton, NJ, 1973. Another excellent book with suggestions for teaching problem solving and with many good problems is: James L. Adams, "Conceptual Blockbusting," W. H. Freeman Press, 1974.

[4] Brainstorming is allowing a class or group of students to respond to a particular question by suggesting ideas which are recorded on the board or on a large sheet of butcher paper. The technique requires that no ideas may be judged, criticized or vetoed until all suggestions have been received and recorded. It encourages participation by all students and leads nicely into discussion of various points of view.

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MATH WITHOUT FEAR PROGRAM AT SAN FRANCISCO STATE UNIVERSITY[5]

Rationale

That large segments of the population fear and dislike mathematics is not a new phenomenon. In the past, this situation has been tolerable because there were sufficient occupational opportunities requiring little or no mathematical skill; however, as our culture becomes more technological and the social sciences more quantitative, an individual who is uncomfortable with basic mathematics finds her/his options severely limited. SFSU is now attempting to help such students to change their attitudes towards mathematics so that they can enter their desired careers.

An experimental course, Math 270, was offered in the Mathematics Department in Spring and Fall, 1976. The course provided positive experiences in mathematics and helped students realize the role fear plays in retarding their progress. Although most students had relied previously on memory to solve math problems, memory, in general, is not a sufficient faculty for learning mathematics at the college level. Consequently this course sought to teach students to "think mathematically" and gain confidence in their ability to do so.

Although Math 270 was quite successful, it was felt by students and the instructor that it could be more successful if it were not taught exclusively in the Mathematics Department, for the following reasons:

- Some students are so afraid of mathematics that they will not enroll in any course in the Department, even if they must abandon their intended major.

- In order for students to speak freely about their fears, they must feel at ease with their fellow students, and it would be easier to have a conducive atmosphere for such discussions if students were more homogeneously grouped.

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[5] Description of the Math Without Fear Program was prepared by Dr. Diane Resek, Project Director. The description is available as a free brochure and may be obtained by contacting her.

- Students with different value systems (e.g., older vs. younger students, minority vs. white students) tended to react differently toward thinking about math instead of learning rote algorithms. It would be easier to develop their problem-solving abilities if they were more homogeneously grouped.

The Classes

With the help of an Innovative Programs grant from the Chancellor's Office, five sections of "Math Without Fear" are being offered in both Fall and Spring 1977-78 in New School, Women's Studies, Education and Mathematics. The success of the program depends upon the selection of instructors who are experienced in this type of course. The instructional staff includes Diane Resek, Project Director, and Daniel Fendel, both of the Mathematics Department, and four instructors from outside the University, Diane Downey, David Ellis, William Finzer, and Rita Liff.

Course Objectives

1. To increase students' ability to reason mathematically;
2. To increase students' enjoyment of and interest in mathematics;
3. To upgrade students' skills in arithmetic and algebra; and
4. To provide students with the rudiments of statistical reasoning.

Course Characteristics include:

1. establishing a non-threatening, non-competitive environment;

2. opening the course with a discussion of "Mathephobia," so students can learn to deal with their fears;

3. using concrete materials and introducing visual models to help explain abstract ideas;

4. presenting students with problems, where they must guess at solutions, so they begin to move away from an exclusive reliance on memory to solve mathematical problems;

5. using logical puzzles to encourage mathematical reasoning in situations that do not appear "mathematical" to students; and

6. helping students learn to read mathematics texts so that they can become more self-sufficient in future mathematics classes.

Mathematics Laboratory

In conjunction with the program, space has been set aside for a mathematics laboratory for use by "Math Without Fear" students, as well as graduates of the course. Three students work in the laboratory. They spend one hour a week in training with the Project Director.

The lab is staffed at other hours by instructors of both the "Math Without Fear" courses and the courses for prospective elementary school teachers. It is open 20 hours a week. The lab features:

1. a comfortable and non-threatening environment where students can study and work together;
2. a supportive tutoring service where students need not fear ridicule and where tutors can learn about students' problems and can experiment with cures;
3. programmed texts and other self-paced material to aid students in remediation and in practicing independence in math learning;
4. various physical materials which serve as models for abstract concepts;
5. math games and puzzles to spark students' interest in math and to entice them to practice mathematical thinking in a context which is less fearful than in conventional subject matter learning; and
6. books and materials with novel ways of presenting math ideas.

Research Component

The program presently deals with four research questions:

1. *Can the one semester "Math Without Fear" course prepare students to succeed in future encounters with mathematics?* This question will be answered by following course graduates for a period of two years.
2. *Do students prefer to enroll in "Math Without Fear" courses in some departments over others?* This question will be answered by polling students.
3. *Are women students more likely to overcome their fears of mathematics in an all-female environment?* Each semester the section of the course in the Women's Studies Department is being restricted to women students. Since instructors visit other classes, they will have a basis for comparison of students' attitudes. This comparison along with student attitude polls should shed light on this question.
4. *Is there an instrument to predict which students will benefit the most from the present curriculum?* In the experimental versions of this course, some students did not move away from a reliance on rote memory by the end of the semester. There is some evidence in the literature that students with different "cognitive styles" will benefit from different teaching approaches. At the end of the first semester we will see if

tests generally used to define cognitive styles will predict which students have learned to think mathematically and which students still rely almost exclusively on rote memory. If a test does have predictive value, students will be placed in sections in the second semester according to their cognitive style. In this event, different teaching techniques will be used in the different sections. The first semester's study will replicate work in this field and the second semester's will extend existing research.

For more information contact:

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Math Without Fear Program
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(415) 469-2251

A BRIEF DESCRIPTION OF THE MILLS PROGRAM TO COMBAT MATH AVOIDANCE[6]

Our feeling is that the best way of overcoming math avoidance is by having students experience success by really "doing it." We believe that attitudes can change rapidly and that several significant positive experiences can make the difference. Thus, our program is set up to get students into a substantive mathematics curriculum quickly, and also to provide our students (even some of the most math anxious) opportunities to become actively involved in (peer) teaching and in (industrial) research projects. Our methods are to set up an environment where it is both desirable and possible to do these things. Necessarily, this program has many facets, including: stimulating interest via publicity, mini-brochures containing non-threatening self-placement quizzes, seminars discussing career options and providing role models; streamlined pre-calculus courses augmented by peer-taught workshops; other goal-oriented workshops including pre-statistics and grad exam workshops; special computer workshops for social science students (e.g., in implementing the SPSS); early and expanded career preparation via industrial internships and dual degree engineering programs.

In particular, the pre-calculus course is specifically designed to prepare students (no matter what their backgrounds) for a calculus sequence in one semester. This is done by streamlining the usual course to include only material directly related to calculus (e.g., graphing the elementary functions), eliminating the remedial aspects (which would just recall negative past experiences) while introducing more advanced, and hence new, interesting and challenging concepts and points of view from calculus (e.g., notions of slope, limit, continuity, smoothness), stressing visualization and developing conceptual skills

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[6] Description of the Mills Program was prepared by Dr. Lenore Blum. She may be contacted for additional copies.

(by learning to ask relevant questions, for example, about symmetry, periodicity, growth properties—and no memorization). The course is augmented by a support structure of peer-taught workshops where students can get individual help in developing supplemental arithmetic and algebraic skills.

We have had considerable success with this project. For example, in the three years since the program began, the number of students enrolled in pre-calculus has tripled (from 27 to 78) and the total number of enrollments in regular math and computer science courses (not including 287 workshop students this year) has doubled (from 331 to 661). Given a steady student population size of 850, these figures are significant.

For more details see: Educating College Women in Mathematics: A Report of an Action Program in Progress, by Lenore Blum in the proceedings of the conference, "Educating Women for Science: A Continuous Spectrum," Mills College, April 24, 1976.

For additional information contact:

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Director, Women in Science and Precalculus Program
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BAY AREA NETWORK FOR WOMEN IN MATH AND SCIENCE: CONFERENCES TO INTEREST GIRLS AND WOMEN IN MATH-BASED CAREERS

In 1975, representatives from projects promoting equal access to math education for women met informally at Lawrence Hall of Science to share ideas. Soon after, another, larger meeting was held at Mills College: Educating Women for Math and Science,[7] under the direction of Dr. Jean Fetter. These early efforts sparked the formation of a network of individuals who have since been meeting regularly at monthly intervals. As a group the network has organized a number of events, primarily conferences to interest women in math and science. What makes the network dynamic and effective is that its members represent many levels of expertise, and new people are always welcome to join. The network includes elementary and secondary school teachers, counselors, university faculty members, representatives from industry, parents, math district people, and students. The combined talents, experience and enthusiasm of these individuals working together have been instrumental in the successful organization of a number of programs.

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[7] "Educating Women for Science: A Continuous Spectrum," proceedings of a conference held at Mills College, Oakland, CA, April 24, 1976.

Conferences sponsored by the network have typically been one- or two-day events. The goals are to make participants aware of the diversity of math-related career opportunities, and to introduce them to successful women working in these areas. Nearly a dozen conferences have been held so far in the Bay Area, attracting 4000 junior and senior high school women as well as university and community women. About 40-60 professional women are invited to interact with participants, usually on a weekend at a college campus. The network provides easy access to women engineers, scientists and technicians, as well as a built-in mechanism for recruiting students via the teachers involved. Attractive brochures are prepared to interest people in attending.

A conference usually begins with a keynote speaker, followed by a panel of women from different fields. The women discuss how they became involved in their fields and describe the challenges and rewards of their profession. Some mention problems they have encountered, often related to their isolation in a male-dominated area. All provide an encouraging note about the variety of options open for women.

During much of the day, small workshops are held in which two or three professionals meet on a more intimate basis with about 15 participants. Some of the workshops are "hands-on". In these, students perform experiments, solve math problems or work with a computer to gain actual experience with the techniques and skills used in these areas. Other workshops are for "career exploration." Here students have a chance to individually question the women about what their work entails, how they feel about their jobs, how much money they earn, what preparation is required, and so on. Since many participants at the outset have little idea, for example, of the diversity of engineering fields, or of what alternative options exist in the health professions other than being a nurse or doctor, they find this quite informative. Participants often comment about how inspiring it is to be surrounded by so many motivated and talented women, who are also willing to take time out to motivate others.

EQUALS: A PROJECT TO PROMOTE SEX-FAIR MATH INSTRUCTION AND COUNSELING

Parents, teachers and counselors strongly influence students' career aspirations and attitudes toward math. At an early age, when girls and boys play with different toys, sex-typing begins. Boys, but rarely girls, are encouraged to take things apart and reconstruct, to invent, to ask questions about what things are made of and how they work. Girls, on the other hand, are taught to depend on males when things need repair or building, rather than to try and figure out much for themselves. These early influences affect the development of risk-taking behavior and problem-solving confidence.which are certainly required for mathematical and scientific proficiency. It is essential that girls and boys growing up today have equal access to experiences which foster development of math and science skills.

To increase awareness of these issues among parents and educators, and to provide viable suggestions for promoting sex-fair counseling and instruction, the Equals Institute, with funding from the U.S. Office of Education, was established at the Lawrence Hall of Science. For three weeks during the summer of 1977, Equals staff members worked with 40 4th-9th grade teachers, counselors and administrators sharing methods for increasing female involvement in mathematics. The participants then assumed the role of educating other teachers, administrators, counselors, parents and students, using the ideas developed.

The areas of emphasis considered important in educating each of these audiences include: expanding awareness of sex-role stereotyping and of limits that girls impose on themselves by not studying math; providing information about the variety of math-related careers and the amount of preparation needed for each; illustrating the impact of role models on creating a realistic picture of possibilities and benefits of entering these careers; stressing the importance of problem solving in mathematics instruction.

Materials developed to accomplish these goals are available and include: "An Annotated Bibliography to Assist Elementary and Secondary School Teachers in Sex-Fair Counseling and Instruction," "An Annotated Bibliography on Classroom Activities for Elementary and Secondary School Teachers to use in Teaching Problem-Solving and Mathematics," and "Instant Investigations" (activities for use in informal classroom 'stations').

A "Teachers' Kit of Equals Activities" is being developed and will be available in the fall of 1978.

To obtain these materials and for additional information contact:

Nancy Kreinberg
Director, Equals Project
Lawrence Hall of Science
University of California
Berkeley, CA 94720
(415) 642-1823

RESOURCE CENTER FOR EDUCATING WOMEN IN MATH AND SCIENCE

One of the most exciting developments in the Bay Area has been the recent creation of a center to coordinate programs that promote participation of women in mathematics and science. With funding for two years from the Carnegie Corporation of New York, the center serves as a clearinghouse of information on projects in the Bay Area and around the country. Co-directors are Nancy Kreinberg, at the Lawrence Hall of Science, University of California, Berkeley, and Dr. Lenore Blum, Head, Department of Math and Computer Science, Mills College, Oakland, California.

Through the center referrals are made and contacts established among program sponsors and interested participants. In addition to disseminating project materials and research data which are currently available, the center is also sponsoring new programs and creating additional resources.

At present several projects are getting started: a conference for adults who wish to establish new programs to interest and prepare women in math is being planned for the fall of 1978, and a handbook is being prepared describing the organization and implementation of the conferences sponsored by the network to interest women in math and science careers.

To reach the center contact:

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Coordinator
Resource Center for Educating Women
in Math and Science
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A CLINICAL APPROACH TO REDUCING MATH ANXIETY

Sheila Tobias

They tell me Newton, Leibnitz and Descartes all invented the calculus or something very much like it at about the same time in history and that's the way I am feeling right now, as one of them. From very different perspectives, Ruth Afflack of Long Beach, Lenore Blum of Mills and the Lawrence Hall of Science people in Berkeley have all really come up with the same kind of intervention. I thought, therefore, that I would emphasize what is different about the project with which I have been associated for three years and particularly about its extension into the adult community.

I start from twin ideological perspectives that are probably different from those of many of the people who are involved in the teaching of mathematics. The two perspectives that energized me at the outset and still guide the projects with which I am associated are the feminist perspective and that of the "math consumer." For those who have not had that heady experience of feminism during the past ten years, let me explain what it was like. Part of the experience of oppression for those women of my generation who were trying to cope with the society we had inherited, was that problems of power were perceived by us to be personal deficiencies. The consciousness-raising process—which has not yet been widely enough understood—did nothing more or less than subject those personal deficiencies to a political analysis from which we could finally understand that they were not deficiencies at all, but rather intelligible responses to an unfavorable situation.

Among those "deficiencies" was what we have come to call "learned helplessness" in certain fields and not in others. The assumption was that women and girls cannot do math because math represents an inappropriate activity for them. Since the belief was that the problem was a personal one, there seemed to be no point in dealing with it constructively. Math anxiety, after this analysis was done, could be seen to be a result of female socialization. We now believe that women have been coaxed into being disabled in math because power is associated with math competence.

The other perspective I bring to this problem is that of a consumer of mathematics. Unlike you, I stopped taking math 25 years ago with trigonometry. I am starting to take math again now. But I have, for the better part of my adult, professional life, opted rather for fields in the humanities. And so you have every right to ask yourselves, what do I have to say to you? What right do I have to participate in this discussion? My answer is that apart from my feminist insights I have very much a right to be here and to be participating, indeed directing, some of these programs, precisely because the consumer, whom I represent, has yet to be heard from. The learner or, in my case, the former learner, the ex-learner, the quitter, may have something to tell you that is worth listening to.

And from these perspectives, let me briefly describe the project that we initiated three years ago at Wesleyan University and which was funded by the Fund for the Improvement of Post-secondary Education and the Sloan Foundation. I must give particular credit to the people who funded us because at the time we came to them we had only an idea. We had no particular direction, but we knew that there were a large number of people at the college level who were avoiding mathematics and we believed we could attract them back into mathematics although we did not yet know how we were going to do this. We asked the funding agency to trust us and they did. Three years and $125,000 later, I think we have something to show for that trust.

First of all, we decided to have a public relations campaign for mathematics as the people at Berkeley and UC Long Beach have done as well. The brochures appended to this paper are the latest in the type we have been distributing since the beginning of our project. Every student in the university receives about six of these a year, and adults in the community get one or two. The purpose of this bombardment is to force everyone to rethink their reasons for avoiding mathematics. Our ideological position is that one has every right to avoid mathematics from strength, but not from weakness. If, after studying our brochure and having an interview with us the individual decides that mathematics is, indeed, not for her, we are not upset. But we insist that people consider taking more mathematics. And we have found that this tactic gets to the heart of the avoidance syndrome because that syndrome involves not only avoiding mathematics but avoiding even thinking about mathematics as well.

The next step for the student who has received this brochure and is wondering what to do next is different from the usual course selection experience in college. The student does not have to decide in the privacy of her room whether or not to sign up for a course. Rather we offer an opportunity to engage in a dialogue about mathematics with our counselling staff. Where we are different from other programs discussed here today is that we have integrated counsellors and math teachers in our math clinic and we invite students and adults to come to us and to talk about math.

The student or adult coming to the clinic will have an entry interview, during which the counsellor will elicit from her what we refer to as a "math autobiography," recollections about school, about parental attitudes, about particular teachers and at the end we ask what was the last mathematical idea the student really understood. This is the moment when trust is very critical, because after a long pause, a student may admit that she never went beyond addition or fractions. This admission is quite comparable to a psychotherapeutic experience where someone will say to a doctor something she has never been able to say outloud before. The counsellor's response is to be tolerant and to say, "Well, then, let's go back and talk about fractions."

For some number of students, indeed for most of them, this one-to-one experience at the entry interview, followed by an exit interview after the course has been taken, is all the counselling they really need. For others, however, this is not sufficient to get them back into mathematics. At the end of the interview, the counsellor and the student will decide on some program: either non-credit modules, two weeks or six weeks in length, or one of the pre-calculus courses we are teaching, or more counselling. Many students simply need more of a buffer and the promise of continuing support before they will enter a math class. And for these, one-to-one counselling can continue as long as needed. Our longest experience was with a student who, you might be interested to know, was not poor in math at all. She was taking a course in linear algebra and doing about B work. Her problem was she couldn't ask any questions in class. She was mute. She dared not even ask her teacher a question after class.

But for most, a few meetings with the counselor followed by a non-credit, non-threatening experience in math are sufficient. Then they graduate to our courses.

What we do in the sub-calculus courses we have introduced is non-radical by intention. We believed that we could not get a sense of the benefit of the counselling dimension if we manipulated too many variables at once. So, essentially we are doing conventional pre-calculus mathematics. We have of course the ideal teaching situation. Our classes are small and homogeneous. We have a very fine, nurturent teaching staff, and the teachers are not above interacting with the counsellors. The counsellors will come into class at their invitation and then give them feedback as to what kind of group dynamics are taking place in the class. By maintaining a normal sequencing of courses and using ordinary textbooks, we feel we can make a judgment about the value of the support system.

As the student goes on into the credit course, he or she knows that there is continued access to the counsellor. It has what the economists call "option value." The fact that the counselling is available, that there is a place to which the student can go to talk about what is happening—on the level of feelings—in math class, is important. Also important is that we emphasize feelings in our teaching. We urge the students to monitor their own feelings, their "self-defeating self-talk" as they learn. This way we believe we can make students ultimately independent of the counselling service insofar as they can begin to counsel themselves.

In working with adults we use a somewhat different technique that for reasons we do not yet understand hasn't worked very well so far with students. It is the equivalent of group therapy. We bring a group of 8 to 12 adults together under the direction of a math teacher and a counsellor and have them share their math autobiographies, while getting to know one another and building up their confidence. During the initial few sessions, the math teacher remains somewhat to the side only participating if mistakes and mistaken notions about math are expressed. After that, some math learning takes place. Since this is never a credit course, the group can determine its own agenda. When the agenda is being set by the group, the dynamic of the class is altogether different. The group may say, "We need more on this," or "Isn't it time to move on?"

Also in this model, we ask the teacher to stop working after no more than 20 minutes so that the counsellor can lead the group through "process." "Process" is a group therapy term that means that the group will go back over everything that happened in the previous 20 minutes and try to talk about the feelings that the learning experience brought forth.

In the book I have just written, Overcoming Math Anxiety, I have a chapter in which I have reproduced almost in novel form the first 20 minutes of such a group session, so that the reader can get a sense of how this operates. In the particular session that I recorded the math teacher became so enthused by the feedback that in his excitement he got carried away. He went to the board and said, "Well, this leads us to the next interesting point. . ." And the counsellor stopped him and pointed out, correctly, that some members of the group had been left far behind.

In terms of evaluation, we are trying to follow up what our students do in the semesters immediately succeeding their experience at the math clinic. Do they select quantitative courses such as chemistry or economics, and do they take more math? Our results so far are between 20 and 25 percent immediate continuation in math or math-related courses. Five hundred and seventy-five Wesleyan students have come through the credit courses of the math clinic since we began, which represents about one-quarter of the student body. More importantly, we know that every one of those students would not have taken college math without the clinic. In addition, 100 students have had counselling and non-credit workshops, and we have processed 75 adults through our group sessions.

What are we learning? Much in the way a mathematician or scientist collects discrete bits and pieces of data and sorts them out looking for patterns, my job has been to look at the math autobiographies and to debrief the teachers and counsellors in order to find answers to the question: What is there about the math anxious person that distinguishes him or her from the math incompetent person? I have written much about this and I am sure that as math teachers you would find all of this important, but in the interest of time, let me mention just a few things to give you a flavor of the difference.

We focus on word problems at the clinic, not only because we believe they are a key element in math learning, but because our math anxious clients seemed to be particularly disturbed by them. Since we have one word problem on the board of our clinic at all times and everybody who comes through the clinic (visitors included) has to attempt to solve it, we have some interesting information as to the varieties of ways in which people try to handle the challenge of an unsolved problem.

One of those problems was the Tire Problem. A car goes 20,000 miles on a trip; the five tires are rotated regularly. The question is: How far will any one tire have gone? You may be interested to know that some people cannot even deal with the question. They think the five tires are all rotating constantly and that the car is driving about three feet off the ground. All that means is that the fact it is a math problem has so traumatized them that they cannot use their good common sense.

After we have helped them understand the language of the problem and have given them lots of time to work at it, we observe how they cope. The math incompetent person will say, typically, "I don't know how to do this." But instead of giving up, he will "poke around at the problem" to see what he can come up with. After solving it he explains that there were three numbers in the problem, 4, 5, and 20,000, so at some point, poking at the problem, he decided to take 4/5 of 20,000 and give the product a "reasonableness test."

Although some math teachers might find this inelegant if not sloppy, we judge the technique to be mentally healthy. The student must be relaxed to be poking around, and to give a tentative answer a reasonableness test is not a bad idea at all. The math anxious person, on the contrary, will not do anything at all. She will simply say, "I can't." Or "This is just the sort of problem I can never do" (before even finding out what kind of work it entails).

In as much time as we give her, she gets nowhere. During the debriefing as we probe to find out if anything useful occurred to her at all, the client may say that indeed the fraction 4/5 did occur to her, or that she realized at some point that 1/5 of the time one tire was in the trunk or that there were 80,000 tire-miles being traveled. But in answer to the question, "Why didn't you pursue this line of reasoning?" or "Why didn't you do the calculation?" she will say, and we have heard this so many times we have to take it seriously, "I figured that if it was in my head it had to be wrong."

And that's it. Or one of the "It's." Math anxious people seem to have no trust in their intuition. Whether this comes from the school experience of not being allowed to guess or because they are girls and are not supposed to do well in math, we do not know. But this is the kind of insight we feel should be brought to the attention of math teachers like yourselves. And that's why I am here.

FACTORS LEADING TO SUCCESS - PRESENT AND FUTURE

Patricia Lund Casserly

It's a heady thing indeed to be given an opportunity to make suggestions in public for other people's future actions. The temptation is strong to assume the solution—to sketch out a utopian program—and to leave the task and implementation to others. But I come to you not as a teacher or a counselor—nor as an administrator—but as a researcher, one who studies the actions and effects of others.

Thus, I cannot claim the experience that flows from the exercise of a responsible post on any of the firing lines of contemporary education, whether in the classroom, the school office or the board room. Yet the vantage point of the researchers has its advantages, too. Few teachers have the opportunities to watch their colleagues at work, let alone to visit other schools in other systems. My work, on the other hand, takes me to many classrooms, serving many kinds of young people, in a rich variety of curricular arrangements in an equal range of circumstances spread across our country. To my constant astonishment, I find in these travels teachers and administrators remarkably ignorant of how curricula and other school practices do differ across this country and, thus, of how different things could be in specific schools if people wished to make them so.

So perhaps I can be helpful by basing my remarks on actions and attitudes I have actually witnessed in real-life schools, and by making suggestions for the improvement, at other, equally real institutions, in the relationship between women and mathematics. In doing so, I intend to foreswear utopian dreams and to confine myself to those things I know are possible, for I have seen them all at work.

The two most recent studies from which my suggestions derive are easy to describe and lead directly to a discussion of my suggestions. The first, supported by the National Science Foundation and the College Board, was based on the observation that preparation for professional careers in mathematics and the physical sciences is, on the whole, open only to those college students who pursued the sustained study of these subjects in secondary school. In it I sought to identify and analyze characteristics of certain secondary schools that have been remarkably successful in attracting and holding young women in strong, sustained programs in mathematics, physics and chemistry. The study examined the curricula, guidance policies, and student cultures of 13 high schools in order to identify those factors that might encourage female participation in advanced high school science and mathematics programs—and to identify those that discourage it.

The second, which is presently being carried out thanks to NIE, is a more focused longitudinal investigation of junior and senior high school mathematics programs emphasizing the identification of attitudinal factors which affect persistence in mathematical study by girls with high ability. Attitudes to be investigated include those of male and female teachers and counselors and those of boys and girls in Advanced Placement mathematics courses. So far, its findings strengthen and enhance findings of the broader study so that the suggestions I should like to share with you today are based on my observations in both these projects.

For the purpose of this paper I'm going to define "success" as "arriving at college with enough mathematics under the belt to step into the full range of college majors without having to clutter up one's freshman year with mathematics courses the college considers remedial." So for most students I'm talking about four years of college preparatory mathematics. But more and more I'm talking about five years of mathematics. That is, sending the girl off to college with a solid introduction to the calculus. My experience is in schools that do just this and do it very well indeed. But the things I have to say about school-related factors that lead to success could apply just as well to schools that do not yet participate in the AP program nor offer their students the calculus.

In order to have a popular, effective calculus class in 12th grade (and let me say that many students study it even a year or two earlier), the school's administration will already have employed a number of tactics designed to encourage girls to stick to the full secondary mathematics curriculum whether it is a four- or a five-year program.

Homogeneous grouping, so accurate and deft that it seems natural to students, is critically important in a subject of sequentially developed skills like mathematics. Bright students in any grade are as unchallenged and hence ill-served by a mixed-ability classroom as regular students would be in a class for slow learners.

The earlier the homogeneous grouping takes place in elementary school, the better it is for girls on the fast track for social as well as academic reasons. It's easier all around if acceleration is an accomplished fact well before sexual self-consciousness and sexual stereotyping are issues—either among peers or among well-meaning counselors. I'll expand on this latter point in a few minutes.

If a girl is persuaded or decides to drop off the track for a year, and many do, there must be a way to climb aboard again. The schools that I've studied make that very provision. They allow and even encourage students to take two mathematics courses concurrently during a single year. Several have summer sessions but, instead of these sessions being primarily or even exclusively for remediation or catching up, they are regarded principally as ways for strong students to accelerate their programs.

Although young women are usually thought of as being several steps ahead of young men in various kinds of maturity, they, like their brothers, often have no fixed idea at grades 8, 9 and 10 in what direction they will go. We are all aware of the phenomenon of the young man "suddenly getting it all together" in grade 11. The problem is that during this time when things are "not together," girls—but not boys—are allowed to drop math since they're getting B's or C's which "aren't too good for the cumulative average" (from the counselor's point of view) or they're "probably going to end up in the social sciences..." (And what a fallacy it is that you don't need mathematics for them!) On the other hand, boys take mathematics all along since "they'll probably need it" in the careers that they, their parents, friends and counselors assume they will have. Yet in these same years girls are trying to arrange these same hopes in the more complex context of marriage and children. At the least, they deserve supportive advice that they not sell their futures short.

In the middle elementary school years (and indeed throughout all schooling) teachers should be aware of other all too prevalent "kindnesses" that are extremely destructive to girls, and are therefore traps thwarting achievement. The first is the "aura-of-the-good-student" trap. Many parents are made aware of this during parent-teacher conferences. The teacher talks about the girl's fine record, how bright and perceptive she is, how well adjusted and how nice to have in class! The parent counters, "Yes, but she can't multiply. Or divide." Or "Manipulating fractions doesn't come easily." And the teacher counters, "Oh, but she's so nice and bright, she really won't have any trouble in school." What the teacher often means is "The girl won't be any trouble." But trouble she will have as will any normal or bright boy who doesn't have his arithmetic skills down cold. I hope that the renewed emphasis on back to basics and the demise of the social pass will help teachers to be really kind by being really firm in insisting that no student slide by him or her without mastery of appropriate arithmetic skills. In high school mathematics it is often the arithmetic skills that hold a student back—causing her to take too long to solve equations she well understands. If you're slow and/or hung up on mechanics, of course you get behind and there's nothing that causes greater anxiety in many of us. This is a great time for a girl to break into tears. Now I'm not suggesting that this is a ploy for sympathy on the girl's part, although it may be in rare instances. It's just that girls are more likely to respond to frustration with tears than boys are. The important thing here, is for the teacher NOT to fall into the "tear trap." Don't respond to the tears, respond to the frustration behind them and try to work the problem through calmly. A number of girls have told me that the supportive calmness of a teacher had saved them from wanting to give up entirely. If the teacher reacts with undue solicitude the girl is apt to receive the message, "My teacher's sorry for me or excuses me because I really can't do this and I cry. Women cry and they really aren't good in math." Nonsense. Women do cry occasionally, even gifted ones, and they can do mathematics, even through tears, but not if the tears fluster those around them or cause mentors to withdraw in embarrassment. Male teachers and counselors who wish to be truly supportive, take note.

And here, perhaps is the place to make a comment or two about women and math anxiety. I'm sure that math anxiety exists, but my research in schools with strong mathematics programs and dedicated, caring teachers does not support the hypothesis that it is more prevalent among girls than boys. And I'm rather sorry that math anxiety has become such a popular slogan for educators and others to bandy about in professional circles and in the media, because it's often used to conveniently lump together all sorts of phenomena associated with learning mathematics or not learning mathematics that aren't manifestations of anxiety at all, at least not at first.

The situation is rather like popular medical writers' recent preoccupation with hypoglycemia as either cause or effect (it was difficult to tell which) of all sorts of pathological living patterns. I think we have to be a lot more scrupulous in our thinking about the relationships of women to mathematics and vice versa. "Math anxiety," like "low blood sugar," is one more faddish, ambiguous term which, sounding insightful and portentious, dissolves upon inspection into a bit of cause, a bit of effect, and much preoccupation on the speaker's part. If we are to use the term, we must surely do so with care, lest it inhibit, rather than enrich and enlighten, our thought.

In the schools of my study, the place I found true math anxiety was not among students but among teachers and guidance counselors. To me this means that mathematics teachers, instead of decrying the products of the lower grades and the advice handed out by counselors, are going to have to take responsibility for the encouragement, retention, and proper placement of young women in the mathematics curriculum. And mathematics teachers are going to have to be responsible for the education of their peers to the importance of mathematics for all students in the world of the seventies and beyond. Let me emphasize this point again in a way that may help to rearrange your thinking or that of someone else with whom you are dealing in your school.

We don't let students drop English because they have trouble spelling, nor because their grammar is imperfect, nor because they don't enjoy the selected literature. Let's not do it in mathematics either. I know the study of one is generally required by law. But there can be a climate for mathematics as there is for English. I've seen it.

Then too, sometimes schools seem to have a good policy for a bad reason. That is, they require that the college-bound student study mathematics at least through the junior year since "students do best on the SAT's if they are studying mathematics concurrently." That's true, but I hate to think that doing well on that test is the only reason some students hear from their counselors for perservering in the mathematics sequence. But such a policy does get a remarkable number of girls "over the hump" who then go on to take the fourth or fifth year. By then, mathematics has become a habit and as the transition to college becomes more imminent, a close reading of catalogues often increases awareness of its usefulness in a wide spectrum of undergraduate fields.

I encourage you and your colleagues to begin to think in terms of a "report card" for your school—particularly your high school. Look at students' math ability by sex in grade 7 or 8 or 9—and in the years thereafter work to see that aqpropriate proportions of these girls—and boys—succeed in your various mathematics sequences or courses. If they don't, make it the business of yourselves, your principals, and your counselors to find out why.

If there are less than representative proportions of girls in upper level mathematics course, your school is not providing appropriate basic education to that segment of its population. We have rarely thought in terms like these but let's begin. Can counselling be improved? You'll probably have to do it! Do the girls need more realistic information on the necessity of math background in a wide spectrum of college majors and careers? You'll have to be the source of that as well. And what about your own attitudes?

Because time is short and I do want to finish, just let me list some other characteristics of mathematics teachers in the schools that are successful in providing appropriate mathematics education for many of their young women as well as for their young men:

1. They use older girls to counsel, encourage and tutor younger girls. An older girl in an advanced math course, several years ahead in school, or in college is often easier for these girls to relate to than a middle-aged "role model" who comes to the school one day to tell young women what it's like to be a "lady engineer" or whatever. I am not belittling the importance for young women of such visitors to the school. I am trying to point out that they are most effective when they are seen as part of a continuum of commitment. Girls, particularly from working class or non-professional middle-class families, want and need to know how one gets from here to there. One girl, on her way to MIT next year, put it this way: "Reading about Marie Curie or other famous women scientists always left me sort of cold. You know, how many men or women win the Nobel Prize? They're impossible for me to identify with. But when a friend of my sister's (a senior in college) came back to school to tell us about what an engineering program was like and some of her experiences in it, ...!" Which leads me to an aside: Those women who are identified by other adults in a school as "good role models" for young women are rarely identified as such by girl students. What girls are attracted to and seek to emulate are teachers or counselors who are seen as trusted older friends, respected mentors, and informed, assertive counselors whatever their sex, marital status and so on.

2. Advanced placement teachers as a group seem to thrive on teaching students, even female students, who may be brighter than they are themselves. This characteristic sets them apart from most of their fellow teachers and is crucial to the young women they teach. For as a rule, gifted young women are not the delight to their teachers that young men are. What is seen as "forthrightness" in boys becomes "rudeness and abrasiveness" in girls. Similarly, "questioning" behavior in boys becomes "disruptive" behavior in girls. And "penetrating insights" become "insolent comments" and so on. Such differential attitudes towards male and female giftedness persist even among those teachers specifically trained to teach the gifted.

3. And third, these teachers have direct access to their students' families and use it. If a girl is having trouble or falling behind, they are able to call the parent directly to discuss

what can be done. They aren't content to let the guidance department handle conferences which may afford them the opportunity of educating the parents of a student of the importance of math for their child.

This has been a necessarily brief and superficial treatment of some of the things that go on in schools that are successful in providing young women as well as young men with the opportunity to take advantage of a full spectrum of curricular choices once they reach college. However, if you adopt some of them in your schools, perhaps when Lucy Sells looks again at the options open to students entering colleges and universities she will not find the horrendous disparity between viable options for young women and young men that she did a few years ago.

CHANGE: THE ROLE OF THE MATHEMATICS EDUCATION COMMUNITY

Judith E. Jacobs

There can be no doubt that the mathematics education community must assume a leadership role in changing things so that girls do not continue to grow up into math avoiders. By the "mathematics education community" I mean those of us who teach mathematics—whether in kindergarten or graduate school, those of us who supervise those who teach mathematics, those of us who as teacher educators prepare individuals to teach mathematics, and those of us who belong to professional organizations concerned with the teaching of mathematics, and, particularly, those recognized leaders of such professional organizations as the National Council of Teachers of Mathematics (NCTM) or the Science and Mathematics Association (SSM). This paper discusses the ways in which the mathematics education community can provide such leadership.

Professional Organizations Concerned With The Teaching of Mathematics

The National Council of Teachers of Mathematics is the largest organization concerned with all aspects of mathematics teaching and learning from the preschool years through the university level. There have been but two surveys to determine the sex distribution of the membership of the NCTM.[1] A 1974 report of the External Affairs Committee of the NCTM states, "The NCTM is the largest professional organization for subject matter in the world with 52,322 individual members. . . However, we know little today about the makeup of this membership." The External Affairs Committee then reported the results of a random sample of 289 members of the NCTM for which there were 222 returned forms. Based on this sample it was determined the membership of the NCTM is comprised of 48.4 percent males and 51.6 percent females. A 1976 NCTM Member Survey, prepared by the Marketing Research Department of the group insurance carrier for NCTM members found that the respondents to the NCTM survey were 49.6 percent males and 40.4 percent female. (The survey was sent to a random sample of 2,251 members and the response ration was 50.1 percent.) One would be conservative, then, to assume that women comprise 50 percent of NCTM's membership and, therefore, women should comprise 50 percent of those assuming leadership roles within the NCTM. A study of the sex distribution of those individuals in such positions found that males have had significantly greater visibility in all areas of participation in NCTM activities at the national level.

Table 1 shows the sex distribution among officers of the NCTM.

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[1] I am grateful to the staff of the NCTM for making the reports and historical records available to me. This facilitated the data collection upon which these analyses are based.

TABLE 1

Sex Distribution in Participation as Officers of NCTM

1920 — 1978

	PRESIDENTS				VICE PRESIDENTS						DIRECTORS					
	Male		Female		Male		Female		?		Male		Female		?	
	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
1920 - 1950	12	66.7	6	33.3	8	27.6	14	58.6	7	24.1	26	43.3	20	33.3	14	23.3
1950 - 1970	9	100	0	0.0	24	55.8	18	41.9	1	2.3	38	73.1	13	25.0	1	1.9
1970 - 1978	3	75.0	1	25.0	-	-	-	-	-	-	26	61.9	15	35.7	1	2.4

A. Number of Participants B. Percent of total ? Initials used or name not identifiable as male or female

Date at left - team begins at the end of the annual meeting

Date at right - team in progress at the end of the annual meeting

Dr. Shirley Hill's team begins at the end of 1978 annual meeting. Therefore, she is not indicated in the count.

In this area there has been only one position in which the number of men who served was not significantly greater ($p \quad .001$) than the number of women who served. This was the position of Vice President. Of particular concern is the disparity in the sex distribution among the Board of Directors for the period 1970-78, for it is from among these individuals that future presidents of NCTM will probably be selected and these are the individuals who approve the members of NCTM's committees and NCTM's representatives to other organizations.

Individual members of NCTM are able to provide leadership by serving on NCTM committees, projects or panels as well as serving as NCTM's representative to other organizations or national committees or boards. Table 2 shows the sex distribution in these positions. The time periods studied reflect the NCTM historical records of such service. The χ^2 analyses of data on just the number of male and female members in these two categories, assuming an expected 50-50 split based on the membership data, indicates that the number of males selected to serve in such positions is significantly greater ($p < .001$) than the number of females. Each category was analyzed separately. One would hope that NCTM's representatives to other organizations and national conferences and boards would reflect the membership of the organization. Yet when NCTM selects someone to represent mathematics teachers it is more likely that a man is selected, thereby perpetuating the myth that mathematics and math teaching are masculine fields of endeavor.

A third area of service, and public visibility, is as a participant on the program of the annual meeting of the NCTM. In considering this data one should remember how important to the professional advancement of many NCTM members such appearances are. This importance is reflected in the tenure decisions at many universities as well as through the informal, and formal, contacts one makes as one becomes known. Table 3 shows the sex distribution in the participation at annual NCTM conventions for every fourth year for the period from 1950 to 1978. The data for presiders, speakers and workshop leaders (1974 and 1978) were analyzed separately. As workshop leaders, the number of women participating was significantly greater ($p < .05$) than the number of men. As speakers and presiders, men appeared on the programs significantly more times ($p < .001$) than women. The 1978 program shows far greater representation of women than have previous programs.

What does this mean? It says that NCTM's annual meetings provide far greater visibility and opportunity for influencing others to male members than to female members. (Remember workshop attendance is limited and attendance at these cost an additional fee.) One could argue that speakers must have specific skills and expertise and the representation on the program represents the available pool of speakers. But how does one explain the significant disparity among presiders?

After studying the data just discussed, one must conclude that NCTM has been a male dominated organization even though its membership is at least 50 percent female. The question is what kind of leadership can this organization give to finding ways of increasing the participation of girls and women in the study of mathematics and in pursuing careers involving mathematics.

TABLE 2

Sex Distribution in Participation as NCTM Committee, Project or Panel Members or as NCTM Representatives

1953 - 1978

	COMMITTEE, PROJECT, AND PANEL MEMBERSHIP						NCTM REPRESENTATIVES					
	Male		Female		?		Male		Female		?	
	A	B	A	B	A	B	A	B	A	B	A	B
1953-1965	504	73.5	164	23.9	18	2.6	51	86.4	7	11.9	1	1.7
1965-1971	557	84.8	92	14.0	8	1.2	63	84.0	11	14.7	1	1.3
1971-1977	474	73.8	158	24.6	10	1.6	77	81.9	17	18.0	0	0.0
1977-1978	167	75.2	55	24.8	0	0.0	19	61.9	9	32.1	0	0.0

A. Number of Participants

B. Percent of Total Number

? Initials used or name not identifiable as male or female

TABLE 3

Sex Distribution in

Participation at Annual NCTM Convention

1950 - 1978

	PRESIDERS						SPEAKERS						WORKSHOP LEADERS					
	Male		Female		?		Male		Female		?		Male		Female		?	
	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
1950							56	60.2	26	28.0	11	11.8						
1954	17	47.2	16	44.4	3	8.3	81	59.6	44	32.4	11	8.0	4	100.0				
1958	23	82.1	5	17.9			49	76.6	13	20.3	2	3.1	1	20.0	4	80.0		
1962	63	73.3	21	24.4	2	2.3	119	73.0	38	23.3	6	3.7						
1966	64	65.3	32	32.7	2	2.0	102	77.3	25	18.9	5	3.8						
1970	88	57.9	59	38.8	5	3.3	173	75.9	46	20.2	9	3.9	6	60.0	4	40.0		
1974	140	60.6	89	38.5	2	.9	244	73.3	76	22.8	13	3.9	61	40.7	79	52.7	10	6.7
1978	119	49.6	114	47.5	7	2.9	372	65.0	185	32.3	15	2.6	98	41.9	120	51.3	16	6.8

A. Number of Participants

B. Percent of Total Number

? Initials used or name not identifiable as male or female

Recommendation

> The NCTM must recognize the legitimacy of the issue of sexism in mathematics education and strive to eliminate such sexism.

Though one must acknowledge the recognition given to this issue through the scheduling of sessions such as these at NCTM meetings throughout the country, more formal attention is needed. Those who attend these sessions already recognize the problem. Somehow those who are not here must be reached. NCTM should develop materials aimed specifically at interesting girls and women in the study of mathematics. NCTM should continue to provide individuals and groups who have developed successful programs with a forum for sharing these ideas.

Recommendation

> The NCTM should seek ways to demystify mathematics and humanize the image of the mathematicians.

Mathematics is viewed as a science which can be mastered by few. Only the very bright are thought capable of doing mathematics. Mathematicians are viewed as aloof, cool and asocial. These views are the antithesis of those values which girls are taught to have. Not only would the attainment of this goal increase the participation of females in the study of mathematics but it would also increase the number of males pursuing mathematical studies. The net result would be an increase in the number of mathematics teachers employed at all levels.

Recommendation

> The NCTM must convince society as a whole that being mathematically competent —"numerate"— is as essential for society today as being able to read.

If more women are to be encouraged to study mathematics they must believe that being mathematically competent is not only desirable but necessary in today's society. This new attitude is one which society in general needs to develop.

Recommendation

> NCTM should seek to increase the participation of women in NCTM activities at the national level.

The data presented is incontrovertible. NCTM has failed to provide equal opportunities for its female members. Specific action needs to be taken to increase the number of women NCTM selects to serve on commitees, panels, and projects, to serve as NCTM's representative to numerous forums, and to participate on the programs of NCTM's meetings. If NCTM is to portray mathematics as a field of endeavor open to all, important to all, and of value to all then those who speak for NCTM should be representative of its membership; i.e., many more women should be selected to speak for the organization.

Similar recommendations should be adopted by other organizations which are comprised of those who teach mathematics or are concerned with teaching mathematics. Some other such organizations are the School Science and Mathematics Association, the Research Council on Diagnostic and Prescriptive Teaching of Mathematics, the Special Interest Group on Research on Mathematics Education of the American Educational Research Association, and state and local organizations affiliated with these groups. The Mathematical Association of America has made strides in this area, in part due to the guidance of the Association for Women in Mathematics.

Teacher Educators Concerned With the Preparation of Mathematics Teachers

Recommendation

> Teacher educators should present models for non-sexist teaching in the mathematics classroom.

For teacher educators to assume such a role it would be necessary for them to become more aware of sexism in the mathematics classroom. This would entail examining the curriculum materials and instructional media used in the mathematics program for sexism. Teachers must not only be aware of this aspect of materials but need to know how to handle sexist materials in the classroom. In addition, the attitudes and perceptions of the teachers regarding the ability and appropriateness of girls and women studying mathematics and pursuing careers involving mathematics should be explored. Teachers are products of society and need the same consciousness raising that society at large needs. Ignoring sexism in the classroom perpetuates it.

Recommendation

> Teacher educators must prepare their students to teach mathematics in non-anxiety provoking ways.

This recommendation has a dual purpose. First, if methods courses are taught in ways which do not produce anxiety among the students, it is likely that these future teachers of mathematics will have better attitudes toward the subject matter. This positive attitude would, in turn, be conveyed to their students. Second, it is important for mathematics to be taught to students in ways which are non-threatening. Only through having experienced learning under such methods can teachers appreciate the importance of how they present mathematics. Teachers tend to teach as they were taught. Therefore, teacher educators must model good mathematics teaching.

Recommendation

> Teacher educators should be certain that their institutions prepare future teachers of mathematics in the subject matter that they will teach.

One cannot teach mathematics in an interesting, exciting way unless one feels competent in one's knowledge of the subject. This can only be accomplished through joint efforts with the NCTM and supervisors of mathematics in seeking to increase the amount of mathematics needed by teachers before they are eligible for certification. Teacher educators must also work closely with college mathematics departments to insure that the mathematics that future teachers study will be helpful to them.

Recommendation

> Both the pre-service and in-service education of mathematics teachers should have a greater emphasis on the applications of mathematics.

Too many mathematics teachers are unaware or unable to identify how mathematics is used. Work in such areas as statistics, computers and applied mathematics could help teachers introduce these ideas into the mathematics curriculum at all levels.

Supervisors of Mathematics

Supervisors of mathematics can provide the teachers and students with whom they work with an atmosphere which is conducive to changing the sexist world of mathematics education. A supportive supervisor who believes that women can teach mathematics as well as men and that girls as well as boys can, and should, learn mathematics can greatly increase the efforts of the classroom teacher in eliminating sexism.

Recommendation

> Supervisors should assign women as well as men to teach advanced courses and honor classes in mathematics as well as to supervise such extra-curricular activities as math teams, math clubs, and math magazines.

Having women teachers in such positions enables them to serve as role models for their students. Seeing a woman teach advanced courses or coach a mathematics team might encourage young women to pursue such activities.

Recommendation

> Supervisors should promote the ideas of employing mathematics specialists in the elementary schools.

According to 1974 data, women comprise 83.3 percent of all elementary school teachers. It is believed that women are more math anxious than men and it is known that women enter college having studied less mathematics than men. One would expect, therefore, that women, i.e., 83.3 percent of all elementary school teachers, may have difficulty in teaching mathematics in a way which is conducive to encouraging their students to continue their study of mathematics. One way of breaking the cycle of having the math anxious teach mathematics is by having specialists teach mathematics in the elementary school.

Recommendation

> Supervisors should encourage the development of early identification programs so that girls gifted in mathematical studies can be encouraged to pursue these talents.

The work of Fox (1976) and Casserly (1975) both indicate that the early identification of these students is likely to increase the chance that they will continue their mathematical studies. Also, early identification enables the girls to form peer groups in which they can find support throughout their precollegiate school years.

Recommendation

> Supervisors should promote the introduction of intervention programs in their schools which have as their goal the increase in the number of girls and young women who continue their study of mathematics.

Intervention programs similar to the ones discussed in previous papers can be located in local schools. In addition, supervisors can join together to develop district-wide programs. Supervisors often have the administrative support which teachers lack to facilitate the development of such programs.

Recommendation

> Supervisors should work towards eliminating the option that all students have to stop their study of mathematics after the ninth or tenth grade.

Girls stop studying mathematics as soon as its study becomes optional. Eliminating or reducing this option will not only increase the number of girls studying mathematics but will greatly enhance the "numeracy" of the general public.

Teachers of Mathematics

Teachers of mathematics are on the front lines in the battle to eliminate sexism in mathematics education. They need the support and help of the others in the mathematics education community. But much of the work in eliminating sexism in the mathematics classroom cannot be mandated or legislated. It must be initiated by the classroom teacher; it must be implemented by the classroom teacher.

Recommendation

> Mathematics teachers should encourage girls to continue their study of mathematics.

All mathematics teachers should be supportive of their female students' interests in mathematics. Women teachers can, in addition, serve as role models. They can demonstrate to their students that mathematics is enjoyable and masterable, that they are mathematically competent and that women can succeed in mathematical activities. Women teachers should offer to teach advanced courses in the mathematics department as well as supervise nonclassroom mathematics activities such as the math team, math club, or math magazine. These activities can be a source of a support group for girls who are interested in mathematics.

Recommendation

> Teachers should make their students aware of the relevance of mathematics to many careers.

If students are to pursue the study of mathematics they need to understand that mathematics is relevant to their future plans. It is particularly important to encourage girls to consider careers which involve mathematics. One way of doing this is by inviting women who have succeeded in careers involving mathematics to speak to your classes, math clubs or at career conferences.

Recommendation

> Teachers must change the view that mathematics is a male domain.

Though the ways in which this myth is challenged may seem small, it is important to attack this view which is pervasive throughout society. One approach is to cease telling jokes or using as illustrative examples those stories which depict women as incompetent in mathematics. You know the kind, the woman who cannot balance her checkbook or figure out the tip. These stories only reinforce stereotypes. A second approach is to screen textbooks and media to eliminate those which perpetuate the stereotype that mathematics is a male domain. Develop a list of materials which girls can use which are supportive of their aspirations. Thirdly, show your students that mathematicians are not asocial and cold. Show the girls that being good in mathematics does not mean that one is not interested in people.

Recommendation

> The mathematics teacher must work with the school counselors to help these counselors understand the importance of mathematics to the futures of their students.

In many schools it is the school counselor who helps students plan their academic studies. Many counselors, consciously or subconsciously, have bought the stereotype that mathematics is a male domain. If these counselors are to be expected to encourage girls to continue their study of mathematics, they must be educated as to the importance of mathematics for career choices as well as be made aware of the sexist bias of many counseling instruments.

Recommendation

> The mathematics teacher must include parental education as one of the means of combating sexism in mathematics education.

Parents need to be made aware of the opportunities from which their daughters will be excluded if they do not study mathematics. Parents should be encouraged to think of their daughters' lives realistically, i.e., it is highly unlikely that their daughters will not work during their adult lives and that the better-paying jobs are in fields which require some study of mathematics.

The preceding recommendations are but some that could be implemented by the mathematics education community. There is much work to be done and the time to start is now.

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MULTIPLE LEVELS FOR CHANGE

Lucy W. Sells

The evidence on sex and race differences in mathematics preparation in high school is overwhelming. As recently as Fall, 1977, 63 percent of the white men entering an East Coast major university had taken three and a half years of high school mathematics (through trigonometry) or more. Their preparation compares with 27 percent of black men, 31 percent of white women, and 19 percent of black women entering with three and a half years or more. Only 9 percent of the white men entered with two years (through geometry) or less, compared with 36 percent of the black men, 46 percent of the white women, and 56 percent of the black women.

In addition, not every school district offers the basic course in trigonometry, required for the standard Freshman calculus course. In California, only 46 percent of the Unified and High School districts offer trigonometry. Whether or not a district offers the course appears to be based on past demand, which has been related to social class composition of students.

Inequality of access to high school mathematics preparation needed for admission to standard Freshman calculus courses requires intervention at multiple levels, rather than remedial work after people are admitted to college. These levels include: the family, the school system, at primary, secondary, and postsecondary levels, the economic system, through potential employers, and the political system, through political decision makers at local, state, and Federal levels.

The Family

Parents need to be adequately informed about the importance of high school mathematics for developing job-related skills for their daughters, as well as for their sons. They need to know that algebra and geometry are as important for access to non-college vocational and technical skills, as trigonometry is for students who plan to go to college. They need to know that average yearly starting offers for students with new Bachelor's degrees are strongly related to the amount of mathematics required in the undergraduate curriculum.

Average yearly starting salary offers in the Spring of 1977 are shown on the following page.

Field	Average Yearly Starting Salary
Engineering	$16,668
Sciences	$13,668
Business and Management	$12,668
Economics	$11,400
Social Sciences	$10,056
Humanities	$ 9,948

Source: Adapted from CPC Salary Survey, March 1978
College Placement Council

Engineering and sciences require the "hard" calculus sequence in the Undergraduate major. Without high school trigonometry, there is no way for the student to complete the undergraduate major in four years. Parents, and students need to know the implications of delaying needed high school mathematics preparation, in terms of additional time and costs of tuition.

Parents and students need to know that job opportunities for people with degrees in Business and Management are good. Further, they need to know that access to these majors requires the "soft" calculus sequence, which also requires trigonometry in high school.

Parents and students need to know the current patterns of employer recruitment. Among the employers recruiting at the University of Maryland in the Spring of 1978, only 16 percent were looking for new employees in undergraduate fields which did not require either the "soft" calculus, or the "hard" calculus. One-third were looking for students with at least the "soft" calculus sequence, and half were looking for students with the "hard" calculus sequence.

	No Calculus	"Soft" Calculus	"Hard" Calculus	Total
Recruiters	33	67	101	201

Source: PLACEMENT MANUAL, Spring, 1978, Office of Career Development, University of Maryland, College Park, pp. 33-43.

The invisible effect of the high school mathematics filter has important implications for equalizing job opportunities for women and minorities. Parents and students should be alerted to these effects. This can be done through organizations such as the Parent-Teacher Association, Girl Scouts of America, and even through a public information campaign, such as a "Smoky the Bear" campaign informing people of the importance of mathematics in keeping career options open.

The Educational System

Teachers, counselors, advisors, administrators, and school board members at primary and secondary levels also need to know all of the above information about the importance of high school mathematics. In addition, they need to know about the highly successful intervention projects that have made a difference in the mathematics achievement of young girls, women, minorities, and even math-scarred elementary and secondary teachers, and counselors and advisors.

It is especially important to address the challenge of transforming mathephobics among classroom teachers and counselors into math-enthusiasts, because that will eliminate a major source of the contagion. It is very difficult for people still experiencing their own math scars to avoid communicating their attitudes to students. It is especially difficult for math-scarred counselors to urge higher level math courses on the students they advise.

One important intervention strategy to break through the pattern of creating math scars at every level is the development of workshops designed to raise the awareness of people teaching mathematics of how their attitude toward the subject, as well as toward students, shapes student performance. In the past, some teachers may have had a tendency to confuse the symptoms of math anxiety, math avoidance, mathephobia, or math bigotry (all terms describing closed mindedness about mathematics) on the part of the student as evidence of "stupidity," or inability to handle the subject matter.

Mathematics faculty at teacher training institutions are likely to misinterpret glazed eyes, frozen, blank faces, and slowness of reaction time as "ineptitude," rather than wondering how they might communicate more effectively. This has important implications for the strengthening of teacher training programs in mathematics for primary and secondary teachers. Teacher training institutions can take greater responsibility for ensuring that the people they train to teach mathematics leave as mathenthusiasts, rather than as mathephobics.

By addressing the problem at multiple levels within the educational system, we should be able to alleviate it within five years. One important measure of our success will be the percentage of women (and minorities) enrolled in high school Algebra I, Geometry, Algebra II, Trigonometry, and in additional-pre-calculus mathematics. The problem will be solved at the point where the percentage of women (and minorities) enrolled in trigonometry and pre-calculus mathematics approximates their percentage in the population of students aspiring to college, and to majors in the calculus-based fields.

For women, the departure from 50 percent enrollment in the advanced mathematics courses is evidence that something else is operating other than equality of social expectations, aspirations, and access to the courses leading to occupational and career opportunity.

One useful measure of the success of Title IX and desegregation efforts is to trace the changes over time in the percentage of women and minorities enrolled in the high school mathematics courses leading to enrollment in a calculus sequence. The departure from trigonometry of large percentages of women and minorities in the population of college-bound seniors suggests something important about the climate of expectations in a given school.

A more refined measure would be to look at the percentage of women and minorities in each successive course: First Year Algebra Geometry, Second Year Algebra, Trigonometry, and Advanced Pre-Calculus Mathematics. The dropoff in percentages of women and minorities from First Year Algebra, through Trigonometry suggests something important about what happens in the mathematics classrooms.

High school mathematics teachers will welcome evidence on the dropoff rate, once they recognize that job security in a time of declining enrollments rests on increasing the supply of women and minorities who are qualified to handle that critical fourth year of advanced mathematics.

The mere fact of asking the question, getting the answers, and disseminating it among studencs, parents, teachers, counselors, and administrators within a district can serve as powerful consciousness raising for people at every level.

The Economic System

Employers need to make clear to school systems, and to parents and students, the kinds of skills needed for upward mobility in the world of work. These skills include the traditional reading, writing, and listening skills, as well as mathematics, science, and technical skills. In addition, employers are looking for a set of attitudes conducive to getting their job done in a work setting. These include some measure of self-confidence and self-esteem, initiative, willingness to take responsibility, and above all, loyalty to the goals and purpose of the organization.

Employers can contribute importantly to strengthening the delivery of mathematics skills, by providing women and minority speakers to serve as role models for junior high school students. The Women and Mathematics Program (WAM), co-sponsored by IBM and the Mathematical Association of America, and the Blacks and Mathematics Program (BAM) co-sponsored by Exxon and MAA, provide a dual function. The intention is to provide role models of people like themselves for students not aware of the opportunities in the math-based careers. An additional side effect is to raise the awareness of white male teachers and students that competence in mathematics transcends sex and race.

When employers, school systems, and families work together in a partnership to equalize delivery of mathematics skills to all students, regardless of race, sex, or social class, we will have created an "everybody win situation."

The Political System

The taxpayer's revolt in California has served notice to political decision makers at local, state, and Federal levels that taxpayers are no longer going to tolerate waste of tax dollars. Inequality of the delivery of mathematics skills, based on sex, race, or social class, is one important measure of the waste of scarce resources. Another is the practice of social promotion of students who have not mastered the skills at one level which are needed to flourish at the next higher level.

The evidence is that students flourish best in a classroom with teachers who are competent in their subject matter, who are enthusiastic as teachers, and who respect their students as competent human beings, regardless of sex or race. In order to respect one's students as human beings, one has to have respect for oneself. What political decision makers at the local level need to know is that teacher self-respect comes from being respected as a competent human being by the administrators who recruit, hire, and promote teachers. The more we find fault and place blame on classroom teachers for the declining test scores, the more we interfere with their effectiveness in the classroom, and force them into a position of finding fault and placing blame on the students, and on the families which send them to school.

What state political decision makers need to know is that a similar process operates to reduce the effectiveness of local decisionmakers in allocating scarce educational resources.

What Federal political decisionmakers need to know is that a similar process operates to reduce the effectiveness of state decisionmakers in allocating scarce educational resources at the state level. Federal decisionmakers have rightfully declared that discrimination on the basis of sex, race, or physical handicap is contrary to public policy.

We could cut through some of the paralysis between the educational and the political system by focusing our energies on producing the results of equal delivery of mathematics skills in the classroom. In the past, too much of our energy has focused on finding fault and placing blame on somebody else, the students, their families, the classroom teachers, the local administrators, "society," television, or local, state, or Federal political decisionmakers. If each of us took responsibility for contributing our own share toward making education work, we could get the job done with less investment of scarce resources.

I'm told by people who work in bureaucracies that 60 to 70 percent of their energy is devoted to covering their tracks from attack from above, and there simply isn't enough time or energy left to do the job. I have not yet had an exception pointed out to me. I suspect that, like human beings, organizations and institutions thrive and flourish best in getting the job done under conditions of autonomy, responsibility, and self-determination.

I've personally experienced two powerful training programs with implications for revitalizing the family system, the educational system, the economic system, and the political system. One is Erhard

Seminars Training (est), and the other is Life-spring. Both training programs come from the position that the world works when we keep agreements, and take responsibility for the way things are. Both have implications for addressing the constraints of belief systems about human beings, based on their race, sex, or position of power and authority, as well as the power or authority system in which they are embedded.

It is gratifying to be able to use the work to equalize access to mathematics skills as a context for the larger work to strengthen the delivery of education in a climate of shrinking resources.

COMMENTS ON THE WOMEN AND MATHEMATICS STRAND
1978 NCTM ANNUAL MEETING

Judith S. Shoemaker

The National Council of Teachers of Mathematics publishes a slick little pamphlet called "Mathematics and My Career" which describes how six people use mathematics on their jobs. This publication is designed for teachers who want to give their students some sense of how mathematics can be used on the job and would be a handy response for those who ask, "Why should I continue to take math? I'll never need it."

Let's suppose that a girl asks that question. The math teacher hands her the pamphlet. As she reads, she discovers something very interesting: "Math must be only for men since this book shows only men who use math on their jobs. I guess I just don't need any more math."

This message, both implicit and explicit, permeates mathematics education, as we can see by the papers presented here—from textbooks and mathematics tests to the teachers themselves and even their professional organization. The message is clear, especially in math-related careers: Math is a male subject, or as Fennema notes, math is a male domain. Boys, not girls, are expected to do well in mathematics by both parents and teachers. Girls who like math and do well in it are not popular with boys and are somehow different from other girls.

Elizabeth Fennema did a fine job at identifying a range of variables that seem to be related to differential course-taking between boys and girls. She begins with an implicit assumption that girls are at least as intelligent as boys and therefore just as capable as boys in achieving well in mathematics. But for some reasons, beginning at about the seventh grade, girls who before this age had been doing at least as well as boys in mathematics, begin to not do as well. It is this age that girls who avoid math most often mention as the last time they understood math. It was fractions, or decimals, or word problems that first confused them and from which they never regained their mathematical "balance." Their confidence is destroyed in their ability to do mathematical problems, their teachers don't expect them to do well, and so they drop it at their first opportunity.

In addition, young girls have very different career orientations from boys. Boys expect that they will have a lifetime at working and they usually settle down early to the fact that they need to prepare themselves for that career. This is not true for girls, even in this liberated society. Although lifestyles are changing and less than half of the mothers in families with children stay home, young girls still truly believe that, for them, they will be wives and mothers first, job holders second. They really believe that their careers won't need mathematics.

As we now know, many women will be pursuing life-long careers, many of which need some competence in mathematics. Many college majors require college algebra or calculus, and graduate studies in the humanities and social sciences require probability and statistics. NIE recently heard from hundreds of young women requesting, and in some cases, pleading for help in increasing their mathematics skills and in improving their confidence in their ability to learn mathematics. These requests came in response to an article in Mademoiselle (March, 1978) magazine called "Math Hang-ups—How to Loose Them."

To most who wrote to NIE for more information math was the critical course that prevented them from completing their college studies. For those returning to college, mathematics was often a roadblock to admission. Others wanting to change their careers were often stymied because of mathematics requirements. The following five letters indicate the range of math-related problems encountered by these young women.

> I have just finished reading "Math Hangups." It was a fascinating article and I learned that I was not the only one who is afraid to tackle a simple math problem.
>
> I enjoyed math all through elementary school and junior high but in high school I had an Algebra and Geometry teacher who screamed at any of us if we missed a math problem. I nearly flunked both courses and have been afraid of math ever since.
>
> Just once I'd love to be able to figure out our joint tax forms or balance our checkbook.
>
> I'd like to learn math all over again.

> I am a substitute teacher in business education and I found the article in Mademoiselle about math anxiety very interesting. I also realize that I fit into that category. Through my four years of college I avoided math as much as possible, taking only what was required.
>
> For the past year, I was considering going back to school for my Masters in Business Administration. But I always hesitated in applying because of the dreaded math requirements. Now I feel that I might have a chance to succeed in this field with your help.

> Thus far I have attempted three years of college and four college-level math courses and have passed a sum total of one. Considering myself a reasonably intelligent person these mathematical pitfalls have resulted in a depression and resignation from college. I feel that my problem is math-oriented since as a pharmacy major at ____________ I understood the theory behind chemistry and physics courses but when it came to practical application I panicked. To date I am contemplating returning to school and a more solid math background might just be the answer. Please send me any available information you have concerning math anxiety.

> I read the article on "Math Anxiety" and very much felt I could identify with this type of anxiety. I am not a poor student but I cannot do well in math. I feel I am a logically thinking person and am told math is logic, and yet I can't see that. I freeze at the thought of going through another math course, and yet I know I have to get through college, and because I did so poorly in Algebra I have to take it again. Is there some way you could possibly help me?

> I desperately need help in overcoming math anxiety because my fear of it is so great that I am at the point of dropping out of college. Currently, I am taking a course in accounting and I feel lost. Not only that, math has been a problem for me ever since ninth grade and my fear grows worse every year. Any information you might have on avoidance programs in __________ would be greatly appreciated. Deep down inside, I really don't want to give up.

What are we doing to help these young women and the future generations of girls who may end up like them? There are many specialized programs in selected areas such as the program at Wesleyan, described by Sheila Tobias. The San Francisco Bay area also has programs aimed at junior high and high school girls. But as we can see from the letters, there just aren't enough of these programs.

The fact that such programs exist is a sad commentary on mathematics education in this country. A large segment of our population (not all women, but a large part of them, plus some men) do not feel comfortable learning mathematics, do not see its relevance to their adult work roles, avoid it at all costs and are therefore cut off from many educational and occupational choices.

We must continue to explore why this is so. At NIE we are sponsoring a $1.2 million two-year research program to find out some of the answers. We are sponsoring ten research grants that are exploring both positive and negative influences on girls' achievement and participation in high school mathematics. These studies are taking a variety of approaches to the problem and include studies of the relationship between spatial-visual skills, sex stereotyping, career aspiration, and mathematics learning. Several of the studies are following girls' decisions over several years to determine when decisions take place. The learning environment itself will be studied by several researchers who are exploring the influence of school and classroom characteristics on girls' achievement and enrollment in mathematics. Several studies are using a longitudinal approach and will try to evaluate the impact of special courses designed to keep girls enrolled in mathematics. Additional studies are taking a retrospective approach and studying adults who have math-related careers.

We are hopeful that the results of this effort, scheduled for completion by December, 1979, will lead to the design of effective classroom strategies that encourage girls to stay enrolled in mathematics. We will devote a year to translating the research into classroom practice. This research along with Jacobs' resolutions should go a long way to improving the participation of women in mathematics.

NCTM, as the major professional association in mathematics education and as the largest professional organization representing teachers, has a special responsibility in the encouragement of girls' participation in mathematics. NCTM should lead efforts to promote the achievement and enrollment of girls in mathematics. The Women and Mathematics strand at the 1978 annual meeting in San Diego represents a recognition of the problem. Now we are ready for some action. A good place to start would be the revision of "Mathematics and My Career."

SOME ADDITIONAL COMMENTS ON THE WOMEN AND MATHEMATICS STRAND, 1978 NCTM ANNUAL MEETING

Dora Helen Skypek

Today's researchers agree that there is no evidence to support the notion that females are less capable of success in mathematics than are their male counterparts. However, the evidence that women are not realizing their capabilities is obvious. A complex collection of sex-related beliefs, attitudes and practices inhibits capable females from following the linear-like sequence in mathematics taken by capable males in secondary schools and colleges. The result is a narrowing of vocational and professional options for women.

What can one do to challenge or to change these inhibiting attitudes and behaviors? In these papers, speakers from the 1978 NCTM program strand on Women and Mathematics describe or recommend a variety of intervention strategies. Afflack describes a course for "mathophobes," adult women who feel a crippling insecurity in mathematics. The course goals include helping "students overcome their fear of mathematics (and) develop their mathematics competency." Tobias discusses a project with similar goals for "math-anxious" women. The latter project includes a group therapy component that helps the participants deal openly with their feelings about mathematics and about their experiences in learning mathematics.

Like Casserly, I am uncomfortable with the phrases "mathophobia" and "math-anxiety" and the attendant implications for women. As with many such labels they tend to divert the attention of the public and the schools from the more basic problems of developmental learning, teacher/counselor attitudes and behaviors, and a school curriculum appropriate for our time. On the other hand, I applaud the efforts that help adult women (and men) to learn mathematics and to change their beliefs about themselves and their capabilities relative to mathematics—and to be quite fair, I can't come up with an alternate label. I view these efforts as interim, or catch-up, measures—along with all the other catch-up efforts of women today.

Liff describes an intervention project for 6-to-14 year-old girls. The out-of-school course, entitled "Math for Girls," is characterized by an environment in which girls experience success in doing mathematics and become aware of the increasing opportunities for women in mathematics. Role models assist in demythicizing the notion that mathematics is exclusively a male domain. This model for intervention may be more difficult to implement than are the ones described for adult women—at least they don't seem to be proliferating to the same extent. However, if "Math for Girls" were copied widely then it could mean that "math avoidance" by mathematics capable women would indeed become a thing of the past.

Nibbelink and Schonberger address the differential influences of mathematics textbooks and tests. Recent efforts to eliminate sex bias from printed materials in mathematics are described. Still further concerns are raised and further research suggested. As one who set herself the task of examining a textbook for sex bias—it took about two hours—I recommend the exercise to every teacher. There is, though, a danger of becoming acutely (even radically) sensitive to the sex-different messages conveyed in verbal communications about users and uses of mathematics.

Casserly identifies observed "real-school" practices that encourage girls to take a full four or five years of rigorous mathematics courses in secondary schools. Among them are (1) homogeneous grouping of students according to ability as early as practicable, (2) catch-up provisions for students who drop off the secondary school mathematics track for a time, (3) encouragement for girls to keep their college major and career options open by taking the full mathematics sequences, (4) calm but firm support from secondary school teachers for girls who want to "give up" when frustrated by skill gaps in arithmetic, and (5) concerned investigation and action if the proportions of boys and girls in the secondary school mathematics sequence do not match corresponding proportions by ability in the earlier grades. She also speaks to the issues of role models, differential attitudes of teachers toward boys and girls who are gifted and the involvement of parents in support of mathematics capable girls.

These practices can be adopted and implemented in nearly all secondary schools. They do not involve extra-curricular intervention efforts and can minimize the influence of sex bias in curricular materials. The practices are those of educators who believe that girls can be as successful as boys, that mathematics is not primarily a male domain. And that brings us back to the importance of attitudes —attitudes of administrators, of teachers, of counselors, of parents, of the girls themselves.

I asked a clinical psychologist how she deals with conflicting attitudes and behaviors of participants in the consciousness-raising groups she directs for decision-makers. She explained that she had given up the therapy, or release-of-feeling, approach. Instead, she provides information about the issue(s) of common concern. She believes that it is information that has impact on attitudes and behavior. The group is provided information about the way things are, the fallacies of certain beliefs, the facts of the problematic situation, the things that individuals and groups can do. A group therapy approach tends to dissipate the anger and anxiety participants feel; they learn to feel better about themselves and others in the group, but little happens outside the group. When, on the basis of information, anger about oppression or discrimination is channelled into goal-specific action, the chances are greater that change will occur in the larger spheres of influence. I think there are implications here for those of us in mathematics education who would change the prevailing practices and beliefs involving women and mathematics.

The research information in Fennema's paper is a beginning place for individuals and groups alike. Fennema reports recent research findings (hers and others), discusses current pervasive myths about sex-related differences in mathematics, and identifies some problems for future research. On the basis of the information reported by Fennema, Casserly, Ernst, Sells and others, Burton challenges teachers of mathematics to examine their own attitudes and suggests a variety of things they can do to bring about change in their own classrooms and schools. Jacobs spells out recommendations for action (1) to The National Council of Teachers of Mathematics, (2) to teacher educators concerned with the preparation of mathematics teachers, (3) to supervisors of mathematics, and (4) to teachers of mathematics.

My own recommendations are two; one is addressed to the readers of this collection of papers and another to the Board of Directors of the National Council of Teachers of Mathematics.

To the readers: Find and organize a group—a PTA study group, a school faculty-staff group, a departmental seminar, a panel/discussion session at your affiliated group meeting, a teacher education or staff development course, a district meeting of curriculum specialists—that is, a grass roots level group. For what purpose(s)? To disseminate the information in these papers and to generate action goals for the individual members of the group and the group itself. Two weeks ago, the mathematics supervisor of a large suburban district called to say, "In this JRME (May, 1978) article on sex-related differences in mathematics, Fennema says that 'important negative influences may exist within the schools themselves.' What is she talking about? And what can I do about it?"

Only then did it occur to me to invite him to our departmental series of open hearings on research proposals and reviews of graduate studies on sex differences in school achievement. The next one scheduled is a study of factors that influence high school girls and boys to enroll in mathematics courses beyond those required for graduation. The subjects of the study are students from his own district. The point is that we must seek out the individuals and group(s) we have access to, get current information (research findings and observed behaviors at the local level) to them, brainstorm possible actions and move on goals that are within our power or the power of the group to achieve.

To the NCTM Board of Directors: Jacobs makes the case that, although 50 percent (or more) of the dues-paying members of NCTM are female, the central positions of power, influence and visibility have gone more frequently to male members of the organization. As she says, "The data are incontrovertible." As a long-time member of NCTM, I admit to being a bit defensive about the data and the conclusions of her study. The uneven distribution of the sexes among NCTM officers, committee members and speakers at national meetings is a reflection of the uneven distribution in the larger population of all educators. At the same time, however, we are forced to recognize yet another symptom of the more basic problem described in these papers.

What I would like to see is public, widely-disseminated recognition by NCTM that the problem of sex-related differences in mathematics is a problem for mathematics educators. Leadership in recognizing the problem, assessing the problem and promoting intervention measures has gone, by default, to other organizations and agencies. Specific guidelines for the elimination of sex bias in curricular materials and research reporting have been established and publicly ascribed to by other educational groups. NCTM has done little more than "go along with" the prevailing trends.

What more can NCTM do? Its Board of Directors and central committees can assume the leadership in fostering among local groups the grass-roots level activities previously described. NCTM can surely do no less than other educational groups; and they can do more by giving encouragement and direction to many of these efforts.

I'm not sure that another central committee is the most effective way to tackle the issues involved. I'd rather see each already established committee re-examine its own policies and responsibilities for purposes of incorporating into its own programming affirmative action to improve the mathematics education for girls and women.

I would like to see NCTM seek financial support for funding mini-projects by individuals and locally-based groups. Dissemination of information should be a priority. Some money for intervention projects such as "Math for Girls" (see Liff's paper) at the elementary and middle school levels could be made available. I am particularly intrigued with Casserly's suggestion of a "report card" for schools. She says, "Look at students' math ability by sex in grade 7 or 8 or 9—and in the years thereafter work to see that appropriate proportions of these girls—and boys—succeed in your various mathematics sequences or courses. If they don't, make it the business of yourselves, your principals, and your counselors to find out why. If there are less than representative proportions of girls in upper level mathematics courses, your school is not providing appropriate basic education to that segment of its population." Could NCTM offer encouragement, and even support "report cards" and remedial action at local levels by local school departments? Also, encouragement and support for a group affiliated with NCTM comparable to those of MAA and AERA (Women Educators and the Special Interest Group/Research on Women in Education) are needed.

We have seen the NCTM Board of Directors assume leadership and give direction to its members in addressing the problems of other subpopulations of students in mathematics—the culturally disadvantaged, the slow learner, the gifted, migrants, urban students. It is high time it also addressed the problems delineated in these papers—problems encountered by girls and women in realizing their capabilities in mathematics.

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